Note Title 8/29/2022

MATH 425 8/29/2022

Notes are posted on my webpage:
www.math.lsa.umich.edu/~ikriz

Igor Kriz

Text: Sheldon Ross : A first course in probability
for this section, ed. 8 or newer

Requirements: 3 Midterm tests, no final

100 pts each In person, in class

tentative dates: Oct 5

Nov 2

Dec 9

HW (total: 100 pts) score = percentage

max. points on the class: 400

HW assigned in each class

typed each Friday on Gradescope

Due by the beginning of next class after it has been posted on gradescope

due also on Gradescope (usually next Monday 10 AM)

this coming week by Wed 10 AM.

Invitation code : 7GEJ35

Office hours MWF 11AM - 12PM or by appointment

Office EH 3846.

A general picture of probability theory

- randomness ?? We do not know what randomness is or if it exists.

- instead of randomness, we discuss <u>measure</u>.

We have a set of outcomes also called a sample space S. An event E is just some subset of S.

In the beginning, we assume S is finite.

To each outcomes, we assign a probability. The probability is a number in $[0,1]$ (between 0 and 1, included). $P(s) \in [0,1]$

The probabilities add up to 1.

sum

$$\sum_{s \in S} P(s) = 1$$

over all events s in S.

is an element of

The probability of an event E is

$$P(E) = \sum_{s \in E} P(s).$$

In many cases, the probabilities of outcomes are determined by symmetries.

<u>Example</u>: Casting a standard cubic die.

Outcomes: $S = \{1, 2, 3, 4, 5, 6\}$

a set of the elements inside the braces

A fair die is symmetrical

equal likelihood of outcomes:

$$P(1) = P(2) = P(3) = P(4) = P(5) = P(6)$$

Therefore, $P(i) = \frac{1}{6}$

for all $i \in \{1, \dots 6\}$.

Example: Suppose I cast two fair dies and I record the two numbers that come up (without remembering which die was which).

same numbers

Outcomes: $\{1,1\}, \{2,2\}, \{3,3\}, \{4,4\}, \{5,5\}, \{6,6\}$

different numbers: $\{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{1,6\}$

$\binom{6}{2} = 15$ $\nwarrow$ soon!

$\{2,3\}, \{2,4\}, \{2,5\}, \{2,6\}$

$\{3,4\}, \{3,5\}, \{3,6\}$

$\{4,5\}$, $\{4,6\}$

$\{5,6\}$

Total of 20 outcomes. <u>Not equally likely.</u>

We can actually instead remember which die was which.

number on die 1   number on die 2

ordered pairs of numbers which came up are symmetrical

die 1 die 2

$P(1, 1) = P(5, 6) = P(6, 5)$

$\vdots$

$6 \cdot 6 = 36$ outcomes equally likely

$P(i, j) = \frac{1}{36}$

From this point of view, unordered pairs are <u>events</u>.

$$P\{i,i\} = P(i,i) = \frac{1}{36}$$

If $i \neq j$

$$P\{i,j\} = P(i,j) + P(j,i) = \frac{1}{36} + \frac{1}{36} = \frac{1}{18}$$

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different   same

$$15 \cdot \frac{1}{18} + 6 \cdot \frac{1}{36} = \frac{30+6}{36} = \underline{1} \quad \checkmark.$$

We have encountered several concepts of probability:

1. mathematical model

   (different sample spaces can describe the same situation, sometimes one is better than the other)

2. counting principles, independent events, etc.

HW: ① Suppose we cast two fair standard dice. Compute the probability that the sum of the numbers that come up is 2, 3, 4, 5, 6, 7, ..., 12. (each of them)

② Suppose we flip three fair coins. Calculate the probability that Heads comes up exactly twice.