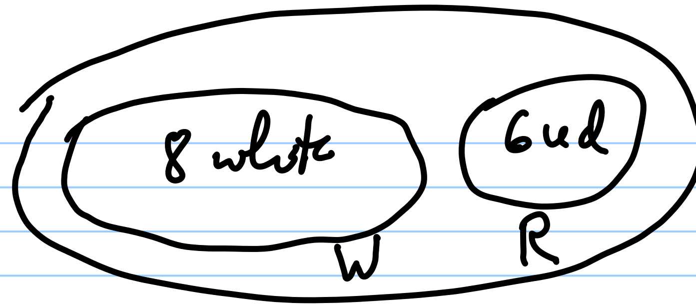
MATH 425 9/9/2022

Note Title 9/9/2022

Miscellaneous probability problems based on finite sample spaces with equally likely outcomes.

Example: A bowl contains 8 white and 6 red balls. If we draw 5 balls randomly (w/o replacement), what is the probability that we draw 2 white and 3 red balls?

Solution:



X = set of all balls

$|X| = 14$

number of elements

S = set of all 5-element subsets of X.

$$|S| = \binom{14}{5}$$

E = those 5-element subsets of X which contain 2 white and 3 red balls

$|W| = 8$

= choosing a 2-element subset of W

and a 3-element subset of R

$$|E| = \binom{8}{2} \cdot \binom{6}{3}$$

$|R| = 6$

$$P(E) = \frac{\binom{8}{2} \cdot \binom{6}{3}}{\binom{14}{5}}$$

$\boxed{\text{"hypergeometric"}}$

<u>Example</u>: Do the same experiment with replacement.

Bowl contains 8 white and 6 red balls.

Choose a ball randomly, record its color then throw it back. Do this 5 times. What is the probability that we have chosen a white ball two times and a red ball 3 times?

Solution: This time, the sample space contains ordered 5-tuples of element of X: $\underbrace{X \times \cdots \times X}_{5} = X^5 = S$

$|S| = 14^5$

E = ordered 5-tuples in S where two entries are in W and three entries are in R

$$\binom{5}{2} \cdot 8^2 \cdot 6^3$$

↑ where did the white ones occur $\left(= \binom{5}{3} \right)$

where the red ones occurred

$$\frac{|E|}{|S|} = \binom{5}{2}\cdot\frac{8^2\cdot 6^3}{14^5} = \binom{5}{2}\cdot\left(\frac{8}{14}\right)^2\cdot\left(\frac{6}{14}\right)^3$$

probabilities of choosing a white resp. red ball in one trial

$\boxed{\text{"binomial"}}$

Example: We have m blue balls and n red balls. We order them randomly. What is the probability of a given order of colors?

Solution: Orders of colors are equally likely.

$$|S| = \binom{m+n}{m} \qquad S = \{\text{orders of colors}\}$$

Answer: $\dfrac{1}{\binom{m+n}{m}}$

Problems involving cards

"Standard card deck" 4 suits ♠ spade

Poker, Bridge ♣ club

(no wild cards) ♡ heart

◇ diamond

52 card total

Each suit contains 13 cards of the following **denominations**:

Jack Queen King Ace

2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

In Poker (most basic rules) you draw 5 cards and you look for certain patterns, which have certain values in the game. General rule: less likely patterns are more valuable.

Straight flush: 5 cards in a sequence in the same suit (A can be counted as the top card or as 1).

Example:

How likely is a straight flush?

Solution: S = { subsets of 5 cards out of 52 }
unordered

$$|S| = \binom{52}{5}$$

E = cards of the same suit,

A 2 3 4 5   7 ⋯ J
2 ⋯ 6   8 ⋯ Q
3   7   9 ⋯ K
4   8   10 J Q K A
5   9
6 ⋯ 10

$$\frac{10 \cdot 4^{\swarrow \text{suites}}}{\binom{52}{5}}$$

<u>Example</u>: How likely is a <u>flush</u>?

= 5 cards in the same suit but not a straight flush.

<u>Solution</u>: $|S| = \binom{52}{5}$

$$|E| = \underset{\text{suits}}{4} \cdot \left(\binom{13}{5} - \underset{\text{straight flushes}}{10}\right)$$

$$P(E) = \frac{4 \cdot \left(\binom{13}{5} - 10\right)}{\binom{52}{5}}$$

HW 7 In a class, there are 10 men and 15 women. A committee of 6 people is chosen randomly. What is the probability that 4 men and 2 women are chosen?

(8) In this class of 10 men and 15 women, people are chosen at random to answer questions (everybody is equally likely to be chosen, regardless of whether they answered a question before). If there are 10 questions,

what is the probability that 5 will be answered by men and 5 by women?

9 The scrabble tiles for "MISSISSIPPI" were scrambled and then arranged randomly in a row. What is the probability that MISSISSIPPI is spelled again?