MATH 425 9/12/2022

Note Title 9/12/2022

Continuing with poker hands:

Straight = 5 cards in a sequence but not a straight flush

$|S| = \binom{52}{5}$

10 sequences of denomination (Ace on top or bottom)

4 suits for each card

$$|E| = 10 \cdot (4^5 - 4)$$ not a straight flush

$$P = \frac{10 \cdot (4^5 - 4)}{\binom{52}{5}}$$

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4 of a kind

4 cards of the same denomination

$$|E| = 13 \cdot 48$$

$$48 = 52 - 4$$

denomination which you have all four cards of

$$P = \frac{13 \cdot 48}{\binom{52}{5}}$$

Full house

3 cards of one denomination
+2 cards of another denomination

$$|E| = 13 \cdot 12 \cdot \binom{4}{3} \cdot \binom{4}{2}$$

which four cards of that denomination you have

choices of denomination

which three cards of that denomination you have

$$P = \frac{13 \cdot 12 \cdot \binom{4}{3} \cdot \binom{4}{2}}{\binom{52}{5}}$$

Three of a kind

Three cards of one denomination and two other cards, but not a full house.

the denomination where we have 3 cards of

not a full house

$$|E| = 13 \cdot \binom{4}{3} \cdot \left[\binom{48}{2} - 12 \cdot \binom{4}{2}\right]$$

two cards in other denominations

which three cards?

$$\binom{4}{3} = 4$$

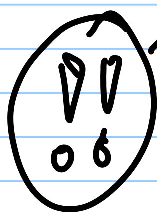
$$p = \frac{13 \cdot \binom{4}{3}\left(\binom{48}{2} - 12 \cdot \binom{4}{2}\right)}{\binom{52}{5}}$$

Two pairs

2 cards of one denomination
2 cards of another denomination
1 other card (of a different denomination of the above two)

✓ the denominations in which you have two cards each

$$|E| = \binom{13}{2} \cdot \binom{4}{2} \cdot \binom{4}{2} \cdot 44 \leftarrow (52 - 2\cdot 4)$$



which cards in the two pairs

$$P = \frac{\binom{13}{2} \cdot \binom{4}{2} \cdot \binom{4}{2} \cdot 44}{\binom{52}{5}}$$

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pair: 2 cards of the same denomination,

3 other cards of different denominations
(and also different denomination
from the above)

denomination of the pair

$$|E| = 13 \cdot \binom{4}{2} \cdot \binom{12}{3} \cdot 4^3$$

which cards in these denominations

which cards of that denomination

three other denominations

$$P = \frac{13 \cdot \binom{4}{2} \cdot \binom{12}{3} \cdot 4^3}{\binom{52}{5}}$$

Card high = none of the above $1 - \sum$ all the above cases

Example: Suppose I draw 6 cards from a standard deck. What is the probability of one pair (and 4 other cards of different denominations)?

Solution: $|S| = \binom{52}{6}$

$$|E| = 13 \cdot \binom{4}{2} \cdot \binom{12}{4} \cdot 4^4$$

denominations of pair

cards in pair

denominations of other cards

cards of other denominations

$$P(E) = \frac{13 \cdot \binom{4}{2} \cdot \binom{12}{4} \cdot 4^4}{\binom{52}{6}}$$

Example: What is the probability of two "threes" when choosing 6 cards in a standard deck?

aaa bbb ↖ denominations

Solution: $|S| = \binom{52}{6}$

$$|E| = \underset{\text{denominations}}{\binom{13}{2}} \cdot \binom{4}{3} \cdot \binom{4}{3}$$

which cards

$$P(E) = \frac{\binom{13}{2}\cdot\binom{4}{3}\cdot\binom{4}{3}}{\binom{52}{6}}$$

A up or down

Example: What is the probability of a sequence of 6 cards (not all in the same suit) when drawing 6 cards in a standard deck?

Solution

$$\left.\begin{array}{llllll} A & 2 & 3 & 4 & 5 & 6 \\ 2 & & & & & 7 \\ 3 & & & & & 8 \\ 4 & & & & & 9 \\ 5 & & & & & 10 \\ 6 & & & & & J \\ 7 & & & & & Q \\ 8 & & & & & K \\ 9 & & & & & A \end{array}\right\} \quad 9 \text{ sequences}$$

$$9 \cdot \left(\underbrace{4^6}_{\text{choosing the suits of my cards}} - \underbrace{4}_{\text{but not ... the same suit for all of them}} \right)$$

$$P(E) = \frac{9 \cdot (4^6 - 4)}{\binom{52}{6}}$$

HW When choosing 6 cards in a standard deck, what is the probability of

① three pair aa bb cc

② four and two   aaaabb

③ flush   6 cards in the same suit, not in a sequence.