MATH 425 09/14/2022

Note Title 9/14/2022

Poker dice (similar questions, but with replacement)

need ordered sample space

cast

5 standard cubic dice

Configurations:

1. 5 alike
2. 4 alike
3. full house (3+2)
4. three alike
5. two pairs
6. one pair

Sample space S: Ordered 5-tuples of $\{1,\dots 6\}$ (7) sequence

$S = \{1,..6\}^5$ $\quad |S| = 6^5$

Probabilities: 5 alike

$|E| = 6$

↗ number on all of the dice

$P(G) = \frac{6}{6^5} = \frac{1}{6^4}$

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4 alike $\quad$ a a a a b ← in any order

$|E| = \binom{5}{4} \cdot 6^{a} \cdot 5^{b}$ $\quad$ $\binom{5}{4}$: 5 choose 4

$$P(E) = \frac{\binom{5}{4} \cdot 6 \cdot 5}{6^5} = \frac{25}{6^4}$$

Full house [a] [a] [a] [b] [b] ← any order

where did a occur

$$|E| = \binom{5}{3} \cdot 6 \cdot 5$$

a b

$$P(E) = \frac{\binom{5}{3} \cdot 6 \cdot 5}{6^5} = \frac{50}{6^4}$$

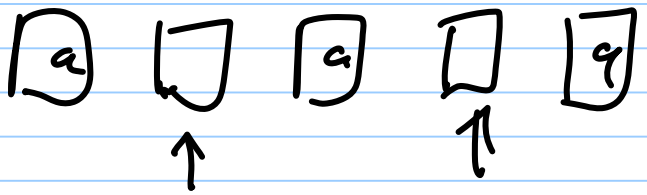
Three alike

← any order

$$|E| = \binom{5}{3} \cdot 6 \cdot 5 \cdot 4$$

↖ where did a's occur

↑ a



↑ 5 choices ↑ 4 choices

$$P(E) = \frac{\binom{5}{3} \cdot 6 \cdot 5 \cdot 4}{6^5} = \frac{200}{6^4}$$

Two pairs [a] [c] [b] [b] [c] any order

$$|E| = \binom{5}{1\,2\,2} \cdot \frac{1}{2} \cdot 6 \cdot 5 \cdot 4$$

a b c

partitioning 5 into three subsets (red, green, blue)

the two pairs are indistinguishable

$$\frac{5!}{2! \cdot 2!} = \frac{120}{4} = 30$$

$$P(E) = \frac{\binom{5}{1\,2\,2} \cdot \frac{1}{2} \cdot 6 \cdot 5 \cdot 4}{6^5} = \frac{15 \cdot 20}{6^4} = \frac{300}{6^4}$$

One pair [a] [a] [b] [c] [d] any order

↙ where is the pair?

a, b, c, d different

$$|E| = \binom{5}{2} 6 \cdot 5 \cdot 4 \cdot 3$$

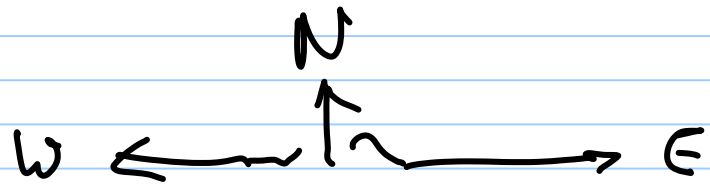
$$P(E) = \frac{\binom{5}{2} \cdot 6 \cdot 5 \cdot 4 \cdot 3}{6^5} = \frac{600}{6^4} = \frac{100}{6^3}$$

Sequence 1 2 3 4 5 or 2 3 4 5 6

any order

$|E| = 2 \cdot 5!$ $\qquad P(E) = \dfrac{2 \cdot 5!}{6^5}$

Bridge is with a standard deck
by four players, divided into two partnerships
NS,
EW.



↓

S

Each player is dealt 13 cards.

Example: In bridge, what is the probability that N has all the spades?

Solution: Choose a good sample space.

S = possible hands of N

$$|\Omega| = \binom{52}{13}$$

$$|E| = 1$$

Answer: $\dfrac{1}{\binom{52}{13}}$

Follow-up: What is the probability that one player has all the spades?

$\leftarrow$ which player

Answer: $\dfrac{4}{\binom{52}{13}}$

Example: What is the probability that N and S have precisely 9 spades between them?

Solution: A good sample space is:

S = hands of N and S put together

$$|S| = \binom{52}{26}$$

E = ($\binom{13}{9}$ their spades) · ($\binom{39}{17}$ their non-spades)

$$17 = \overset{\lambda}{26} - 9$$

$$P(E) = \frac{\binom{13}{9} \cdot \binom{39}{17}}{\binom{52}{26}}$$

Example: What is the probability that E and W have A, K, Q, J, 10 of diamonds together?

Solution: S = Joint hand of the partnership E + W

52-5 $\searrow$ $|\Omega| = \binom{52}{26}$

$|E| = \binom{47}{21}$

$\nwarrow$ 26-5

$\overbrace{AKQJ10\diamondsuit}^{5}$ = specific cards

Answer: $\dfrac{\binom{47}{21}}{\binom{52}{26}}$

Follow-up: What is the probability that E+W have AKQJ10 of diamonds together, and precisely 9 diamonds altogether?

Solution: Ω = cards of E+W together

$$|\Omega| = \binom{52}{26}.$$

Event $|E| = \binom{8}{4} \cdot \binom{39}{17}$ 5 cards are fixed

$26 - 9 = 17$

non-diamonds

$39 = 52 - 13$

4 diamonds out of the remaining 8 diamonds

Answer: $\dfrac{\binom{8}{4} \cdot \binom{39}{17}}{\binom{52}{26}}$

HW ④ On casting 6 standard dice, what is the probability of 3 pairs?

[a] [a] [b] [b] [c] [c] a, b, c different any order

(5) On casting 4 standard dice, what is the probability of a sequence?

[a] [a+1] [a+2] [a+3] any order

(6) In bridge, what is the probability

of one (any) player having precisely three aces?

(7) In bridge, what is the probability of N & S having exactly 8 hearts, including AKQ of hearts?