MATH 425 9/16/2022

Note Title 9/16/2022

<u>Example</u>: A standard poker deck is shuffled. We turn over one card at a time until the first A appears. Is the next card more likely to be the $A\spadesuit$ or $2\clubsuit$?

<u>Solution</u>: Sample space are all shuffles:

$$|S| = 52!$$

Let C be any card. E_C = C is ~~after~~ well after the first ace.

$$|E_C| = 51!$$

To count the outcomes in E_C: ① take C out of the deck. ② Shuffle the deck w/o C. ③ Place C after the first A.

$51!$

$$P(E_C) = \frac{51!}{52!} = \underline{\underline{\frac{1}{52}}}$$ (it does not depend on what card C is!)

$$P(E_{2\clubsuit}) = P(E_{A\clubsuit}) = \frac{1}{\Omega}.$$

Example: A football team has 20 offensive and 20 defensive players. During a road game, they stay in a hotel, 2 players per room. If they are assigned at random, what is the probability that every offensive player is paired with an offensive player?

Solution: Choose the sample space.

Book solution: All possible matchings (w/o keeping track of which room they are in)

$$|S| = \frac{\binom{40}{\underbrace{2\ 2\ \cdots\ 2}_{20}}}{20!} = \frac{40!}{20! \cdot 2^{20}}$$

$$|E| = \left(\frac{\binom{20}{\underbrace{2\ 2 \cdots 2}_{10}}}{10!} \right)^2 = \left(\frac{20!}{2^{10} \cdot 10!} \right)^2$$

$$P(E) = \frac{\left(\frac{20!}{2^{10} \cdot 10!} \right)^2}{\frac{40!}{2^{20} \cdot 20!}} = \frac{(20!)^3}{10!^2 \cdot 40!}$$

Method 2 : Room numbers : 1 - 20.

Which players are in which room?

$$|S| = \binom{40}{\underbrace{2 \cdots 2}_{20}} = \frac{40!}{2^{20}}$$

$$|E| = \binom{20}{\underbrace{2 \cdots 2}_{10}}^2 \cdot \binom{20}{10} = \frac{20!}{2^{20}}^2 \cdot \frac{20!}{(10!)^2}$$

which rooms
are the offensive players
staying in 2

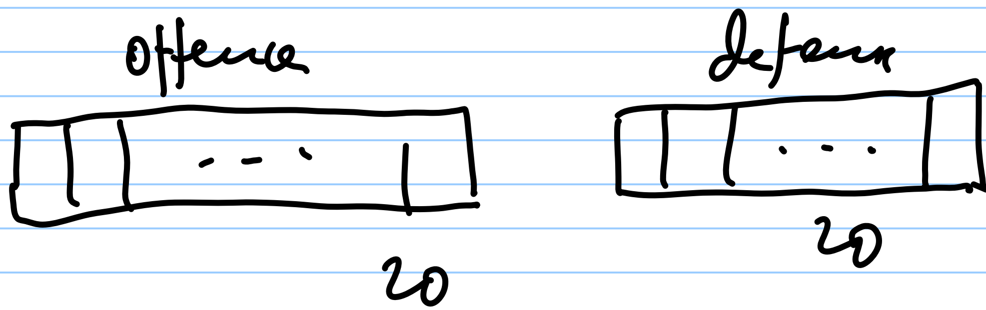
$$P(E) = \frac{\frac{20!^2}{2^{20}} \cdot \frac{20!}{(10!)^2}}{\frac{40!}{2^{20}}} = \frac{20!^3}{40! \cdot 10!^2}$$

<u>Follow-up</u>: What is the probability that all offensive players <u>except k</u> are bound with offensive players?

<u>Solutions</u>: Before use the sample space

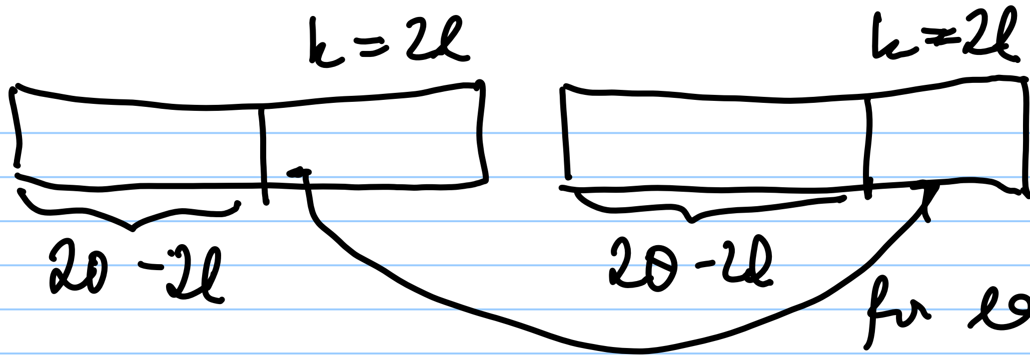
w/o room numbers.

$$|S| = \frac{\binom{40}{2 \ldots 2}}{20!} = \frac{40!}{20! \cdot 2^{20}}$$



E_k : all offensive players <u>except k</u> are housed with offensive players

<u>$P(E_k) = 0$ if k odd</u>



for each offensive player among the last 2l pick a defensive player among the last 2l

$$|E_k| = \left(\frac{\binom{20-2l}{2 \cdots 2}}{(10-l)!} \right)^2 \cdot (2l)!$$

$$P(E_k) = \frac{\left(\frac{(20-2l)!}{2^{10-l} \cdot (10-l)!} \right)^2 \cdot (2l)!}{\frac{40!}{20! \cdot 2^{20}}}$$

$$\boxed{\dfrac{\left(\dfrac{(20-2l)!}{2^{10-l}\cdot(10-l)!}\right)^2\cdot(2l)!}{\left(\dfrac{40!}{2^{20}\cdot 20!}\right)}}$$

Why do we need to look beyond finite sample spaces with equally likely events?

Example: Keep flipping a fair coin. What is the probability that H come up first on the nth try?

Solution E_1 : H on first try $\quad P(E_1) = \frac{1}{2}$.

E_2 : H on second try $\qquad P(E_2) = \frac{1}{4}$

TH

$\vdots$

E_n : H first on nth try

$\underbrace{T \cdots T}_{n-1} \underset{1}{H}$ $\qquad P(E_n) = \frac{1}{2^n}$

Observe that in the right mathematical model, there are infinitely many outcomes

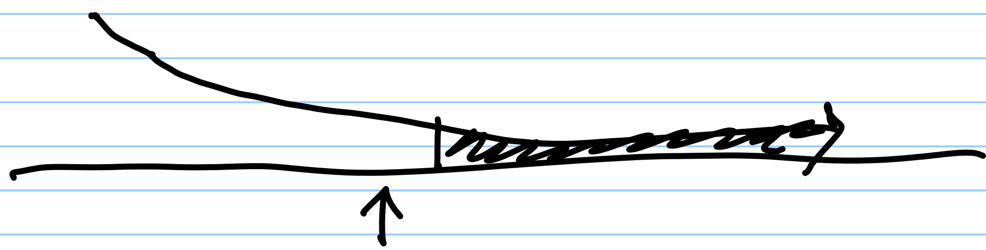
$S = \{1, 2, 3, 4, \ldots\} \cup \{\infty\}$

$$\underbrace{P(E_\infty)}_{\text{H never come up}} = 1 - \overbrace{\underbrace{\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots\right)}_{1}}^{\text{H never come up}} = \underline{\underline{0}}$$

<u>Example</u>: What is the probability in a certain location that there will be no earthquake in the time interval $[0, t]$?

(Not enough information)

Imagine a coin ~~is~~ being flipped at each moment of time which comes up H with a certain probability rate.



the probability rate that it happens at time t and not before decreases exponentially.

We need to reexamine our concept of probability to handle infinite models "discrete" and "continuous".

HW ⑧ On a road trip of the football team with 20 offensive and 20 defensive players, they are accommodated in 20 rooms (no. 1-20), 2 per room (randomly). What is the probability that all offensive players are in rooms 1-10?