MATH 425 9/23/2022

Note Title 9/23/2022

$X_1, X_2, \ldots X_n$

$\bigcup_n X_n = \{ x \mid \text{for at least one } n,\ x \in X_n \}$

union

$\bigcap_n X_n = \{ x \mid \text{for all } n,\ x \in X_n \}.$

intersection

Example: Suppose you got the full score 100 on a standardised test. The results from the testing company say that you placed at the 96 percentile. What does this mean?

Answer: 4% for all the test-takers of this test also got the full score.

100 = top score

$$0.96 = p(\text{score} < 100) \quad \Big\} \quad P(\text{score} \le 100) = 1$$

score is not a continuous random variable

(In probability, we will study random variables which are functions X on the sample space such that $P(\underbrace{X \le a}_{\text{measurable}})$ exists.

this is the cumulative distribution of X.

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set of all people whose score is < 100 is the union of the set of people whose score is $\le a$ for all $a < 100$. ⌋

Let us suppose that the score on the test could be any real number between 0 and 100.

approach 100 arbitrarily closely

$$[0, 99] \cup [0, 99.5] \cup [0, 99.75] \cup \cdots$$

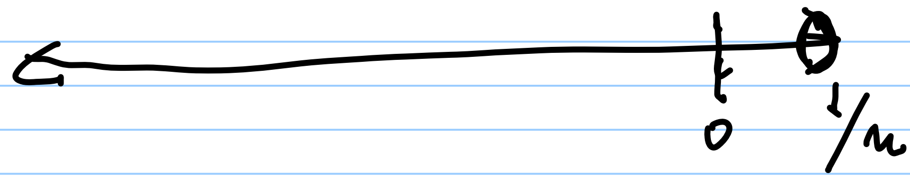
$$\cup \left[0, 100 - \frac{1}{2^n}\right] \cup \cdots$$

$$= [0, 100)$$

an example where the union of countably many closed intervals is (half) open.

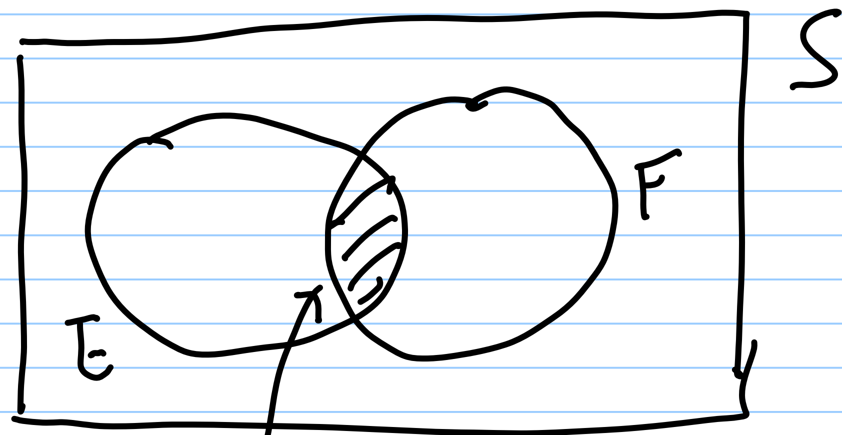
Example: $\bigcap_{n=1,2,\dots} \left(-\infty, \frac{1}{n}\right) = (-\infty, 0]$

open — 0 included.



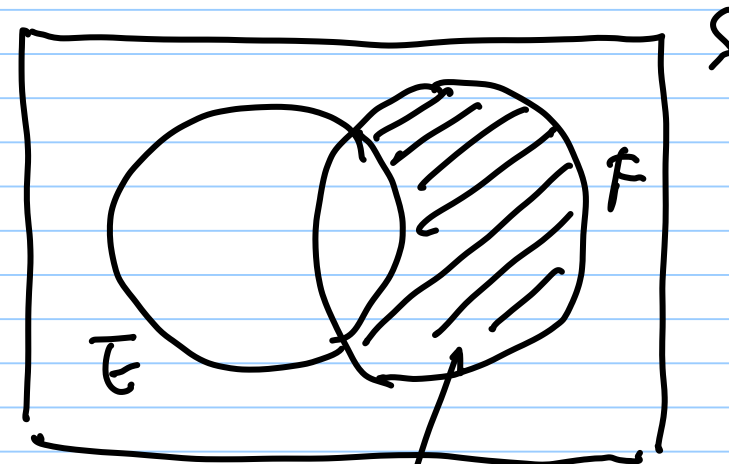
Set operations with finitely many sets

Venn diagrams (work best for at most 3 sets)



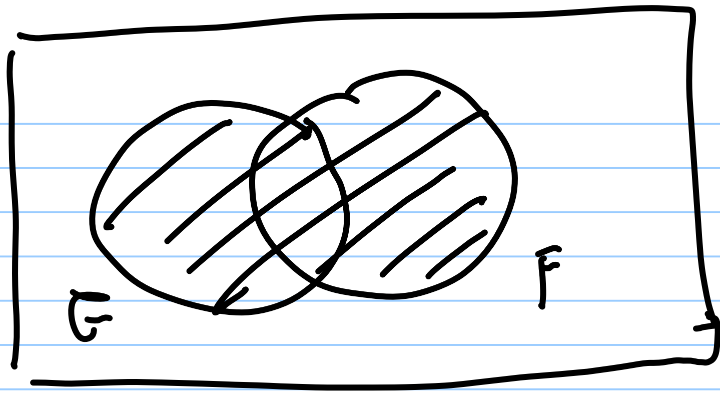
$E \cap F = \overbrace{E \cdot F = EF}^{\text{Ross}}$

intersection



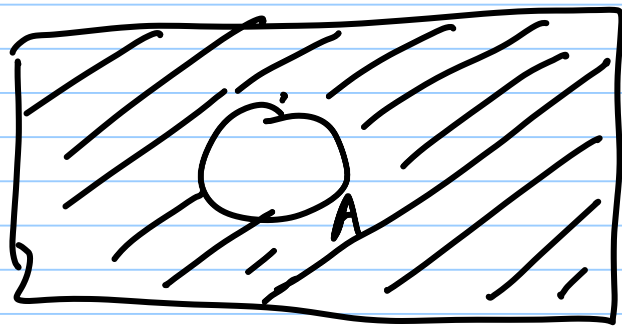
$F \setminus E$

"set theoretical difference"



$E \cup F$

union



$\setminus A = S \setminus A$

$\parallel$

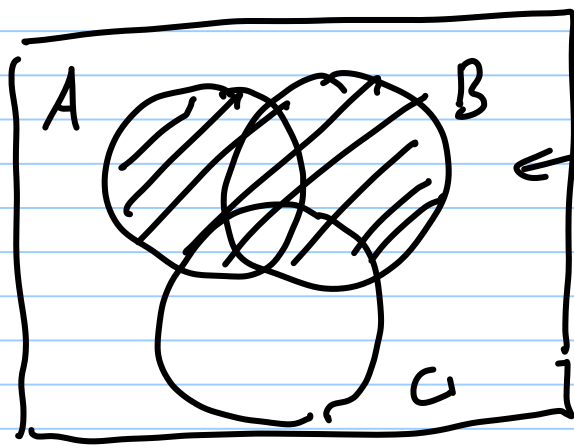
A^c

sample space

S

Example: Prove by Venn diagram that

$$(A \cup B) \cap C = (A \cap C) \cup (B \cup C).$$

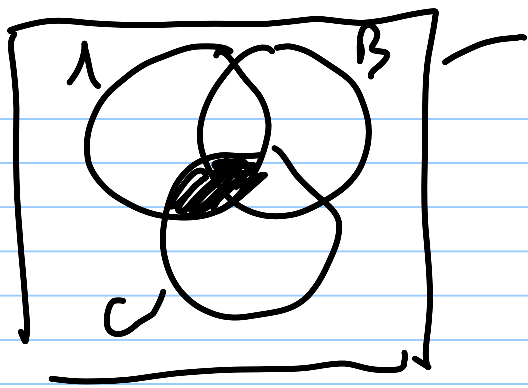


Venn diagram of three sets

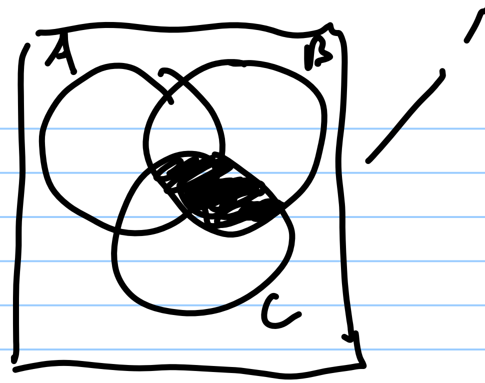
$A \cup B$

$(A \cup B) \cap C$

$(A \cap C) \cup (B \cap C)$



$A \cap C$

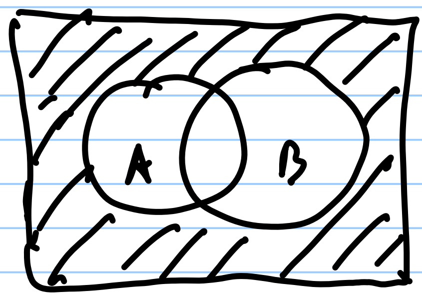


$B \cap C$

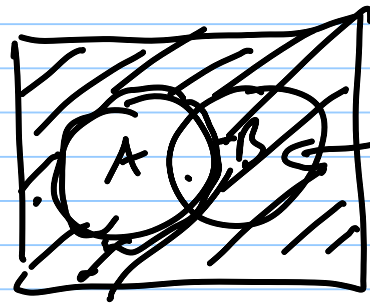
The regions are identical and that proves the identity.

De Morgan rule : unions and intersections vs. complements (they get <u>reversed</u>)

$$(A\cup B)^c = A^c \cap B^c \qquad (A\cap B)^c = A^c \cup B^c$$



$(A\cup B)^c$



A^c

$B\setminus A$

$A\setminus B$

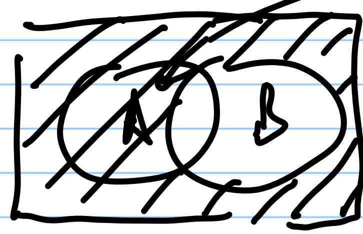
appear only in one picture

disappear when taking

$A^c \cap B^c$

so we get

$(A\cup B)^c$.

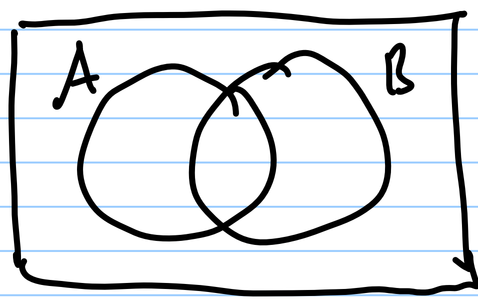


B^c

$$\left(\bigcup_n A_n\right)^c = \bigcap_n (A_n^c) \qquad \left(\bigcap_n A_n\right)^c = \bigcup_n (A_n^c)$$

$$A^{cc} = A.$$

Using Venn diagrams with probability:



$$P(A \cup B) = \overbrace{P(A)}^{P(A\cap B)+P(A\setminus B)} + \overbrace{P(B)}^{P(A\cap B)+P(B\setminus A)} - P(A\cap B).$$

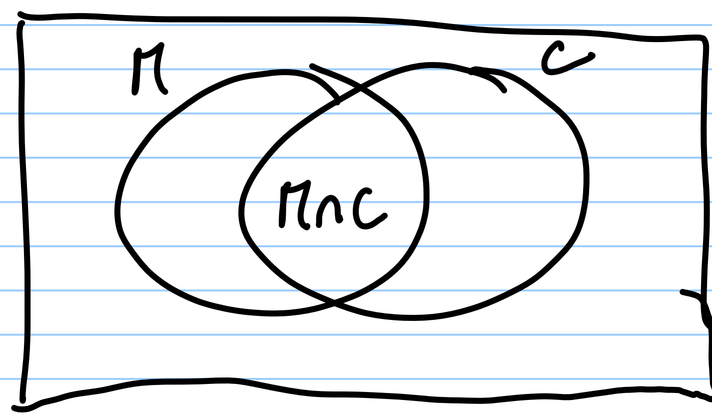
Inclusion and exclusion principle for two sets

Example: The probability of scoring at least 60% on a Math test is 0.9, the probability of scoring at least 70% on a Chem. test is 0.8 and the probability of scoring $\geq 60\%$ on the math and $\geq 70\%$ on the chem. is 0.75.

What is the probability that you score $\geq 60\%$ on the math or $\geq 70\%$ on the chem?

meaning and/or
(one or the other or both)

Solutions:



M = Math ≥ 60%

C = Chem ≥ 70%

$P(M) = 0.9$

$P(C) = 0.8$

$P(M \cap C) = 0.75$

$P(M \cup C) = 0.9 + 0.8 - 0.75 = \underline{\underline{0.95}}$

HW 6 $\bigcap_{n=1,2,\ldots} \left[-1-\frac{1}{n},\ 1+\frac{1}{n}\right] =$ ← (which interval?)

7 $\bigcup_{n=1,2,\ldots} \left[-1+\frac{1}{n},\ 0\right] =$

8 The probability of a college student majoring in math is 0.2, the probability of

majoring in chemistry is 0.3, the probability of majoring neither in math nor in chemistry is 0.6, What is the probability of double-majoring in math and chemistry?