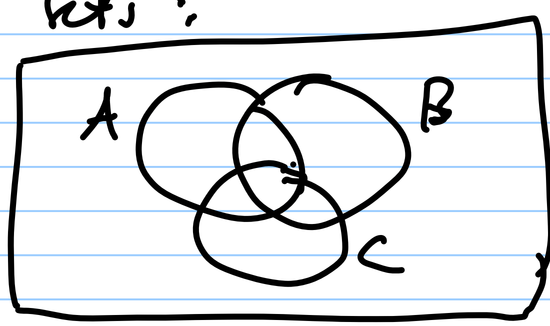
MATH 425 9/26/2022

Note Title 9/26/2022

Inclusion and exclusion principle for 3 or more sets (version for counting measure probability)

The case of 3 sets:



Probability version

$$\boxed{P(A\cup B\cup C) = P(A)+P(B)+P(C) - P(A\cap B) - P(A\cap C) - P(B\cap C) + P(A\cap B\cap C)}$$

How many times $P(A\cap B\cap C)$ was counted

$P(A) + P(B) + P(C)$ $\quad$ $P(A\cap B) + P(A\cap C) + P(B\cap C)$

$3 - 3 + 1 = 1$

In the general case of n sets

$$P(A_1 \cup \cdots \cup A_n)$$

can be counted by <u>summing</u> the probabilities of intersections of <u>odd numbers</u> of the sets $A_1, \dots, A_n$ and <u>subtracting</u> the probabilities of intersections of <u>even numbers</u> of the sets $A_1, \dots, A_n$

$$P(A_1 \cup \dots \cup A_n) = \sum_{k \geq 1} \sum_{1 \leq i_1 < \dots < i_{2k-1} \leq n} P(A_{i_1} \cap \dots \cap A_{i_{2k-1}})$$

$$- \sum_{1 \leq i_1 < \dots < i_{2k} \leq n} P(A_{i_1} \cap \dots \cap A_{i_{2k}})$$

minuses

How many times do we count

$P(A_1 \cap \cdots \cap A_n)$?

$$\binom{n}{1} - \binom{n}{2} + \binom{n}{3} - \cdots - (-1)^n \binom{n}{n} =$$

intersections of 1 set — intersections of 2 sets — intersections of 3 sets

$$\overbrace{\binom{n}{0}}^{1} - \binom{n}{1} + \binom{n}{2} - \cdots + (-1)^n \binom{n}{n} = 0$$

$$\binom{n}{1} - \binom{n}{2} + \cdots - (-1)^n \binom{n}{n} = \underline{\underline{1}}$$

$$(a+b)^n = \overbrace{a^n \binom{n}{0} + a^{n-1} b \binom{n}{1} + \cdots + b^n \binom{n}{n}}$$

$$a = 1 \qquad b = -1$$

Example (3 sets): Three events E, F, G have the following probabilities:

$$P(E) = \frac{13}{100}, \quad P(F) = \frac{15}{100}, \quad P(G) = \frac{21}{100}$$

$$P(E \cap F) = \frac{3}{100}, \quad P(E \cap G) = \frac{6}{100}, \quad P(F \cap G) = \frac{7}{100}$$

$$P(E \cap F \cap G) = \frac{2}{100}$$

Calculate $P(E \cup F \cup G)$.

Solution:

$$P(E \cup F \cup G) = \frac{13}{100} + \frac{15}{100} + \frac{21}{100} - \frac{3}{100} - \frac{6}{100} - \frac{7}{100} + \frac{2}{100} = \frac{35}{100}$$

Follow-up: What is the probability that none of the events E, F, G will occur?

$$1 - \frac{35}{100} = \frac{65}{100}$$

Other versions: Counting: S = 100 college students

E = taking Math

F = taking Physics

G = taking Chemistry

Number of student taking

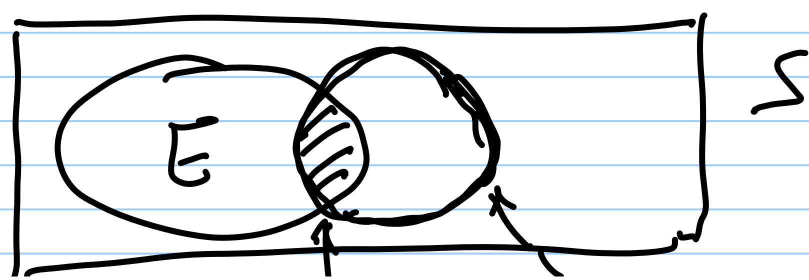
Math	13
Physics	15
Chem.	21
Math and Phys	3
Math and Chem	6
Phys and Chem	7

Math, Phys, or Chem 2.

How many students take one or more of the subjects Math, Phys or Chem?

$13 + 15 + 21 - 3 - 6 - 7 + 2 = \underline{\underline{35}}$

Conditional Probability



The probability $E \cap F$ only in the new sample space F is called **conditional probability**

I want to restrict to a <u>what</u> sample space corresponding to an event F (only interested in outcomes in F)

$$\boxed{P(E|F) = \frac{P(E \cap F)}{P(F)}}$$

$$\left(P(F) \neq 0 \right)$$

<u>Example</u>: The probability that a person will live to ~~the~~ the age of 60 is 0.8. The probability

that a person will love to the age of 80 is 0.5. What is the probability that a 60 year old person will live to the age of 80?

<u>Solution</u>: F = person loves to the age of 60

E = person loves to the age of 80,

In this scenario,

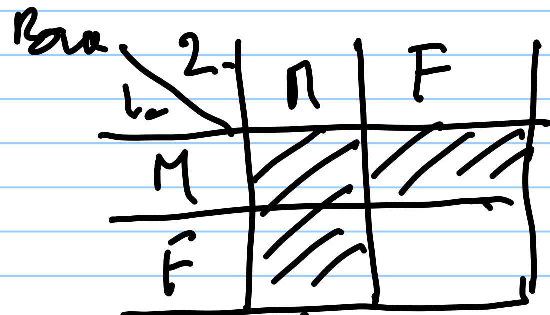
$E \subseteq F$

$$E \cap F = E$$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{P(E)}{P(F)} = \frac{0.5}{0.8} = \underline{\underline{0.625}}$$

Example: Suppose a person has two children and we know at least one of them is biologically male. What is the probability that both are biologically males?

Solution:



both males

$$P(E) = 1/4$$

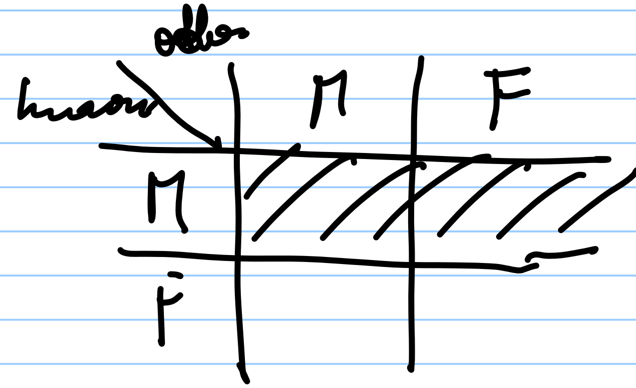
Conditioning events: Φ

$\geq$ One is male

$$P(\Phi) = \frac{3}{4}$$

Again, $E \subseteq \Phi$: $P(E \mid \Phi) = \dfrac{1/4}{3/4} = \underline{\underline{1/3}}$

<u>Follow-up</u>: A person has two children. I know one of them and they are biologically male. What is the probability the other one is biologically male?



$$\frac{1/4}{1/2} = \underline{\underline{\frac{1}{2}}}$$

<u>Answer</u>: $\underline{\underline{\dfrac{1}{2}}}$.

HW ① At a certain university, the probability of a student taking English (E), History (H), Political science (P) and combinations of the subjects are

$$P(E) = \frac{1}{2} \quad P(H) = \frac{1}{3}, \quad P(P) = \frac{1}{4}$$

$$P(E \cap H) = \frac{1}{6} \quad P(E \cap P) = \frac{1}{8} \quad P(H, P) = \frac{1}{12}$$

$$P(E \cap H \cap P) = \frac{1}{24}.$$

What is the probability that a student takes none of these subjects?

(2) Suppose that a certain car rental company only buys blue and red compact cars. 2/3 of their cars are blue and 1/3 are red. A branch location has two compact cars. (a) If I know that

at least one of them is blue, what is the probability that both are blue?

(b) If I saw a blue compact car drive off their lot, what is the probability that the other one is blue?