MATH 425 9/28/2022

Note Title 9/28/2022

Test 1 : A week from today in class

50 min exam, 5 questions, each worth 20 pts.
(can be subdivided into parts). No calculators.
Answers should be expressions
(unless specified), legible

Topics : on Friday (everything before conditional probability)

On Monday Review.

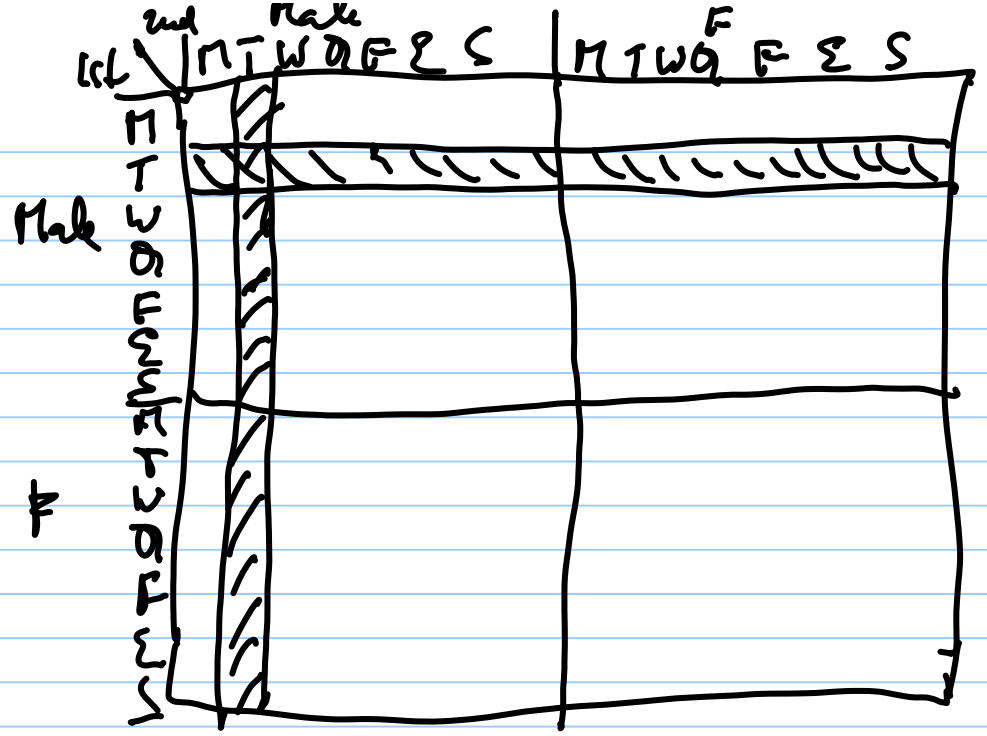
Accommodations: You need to schedule your exam with TAC. (must start at the same time to get the exact same test)

Back to conditional probability

Example: Suppose a person has two children one is a male born on Tuesday. What is the probability they are both male?

└at least

Solution:



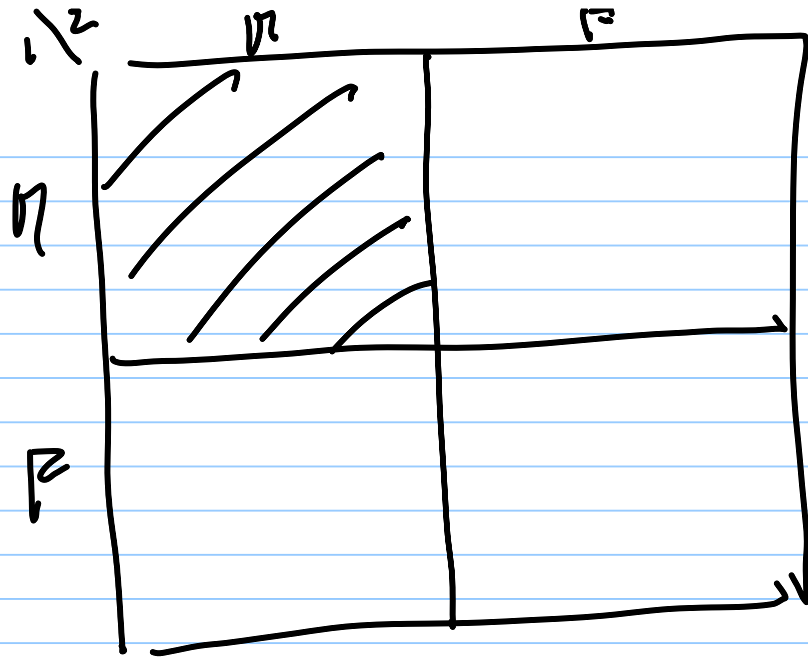
T: Condition on event: at least one is a male

Leave on T

$$P(T) = \frac{14+14-1}{14^2} =$$

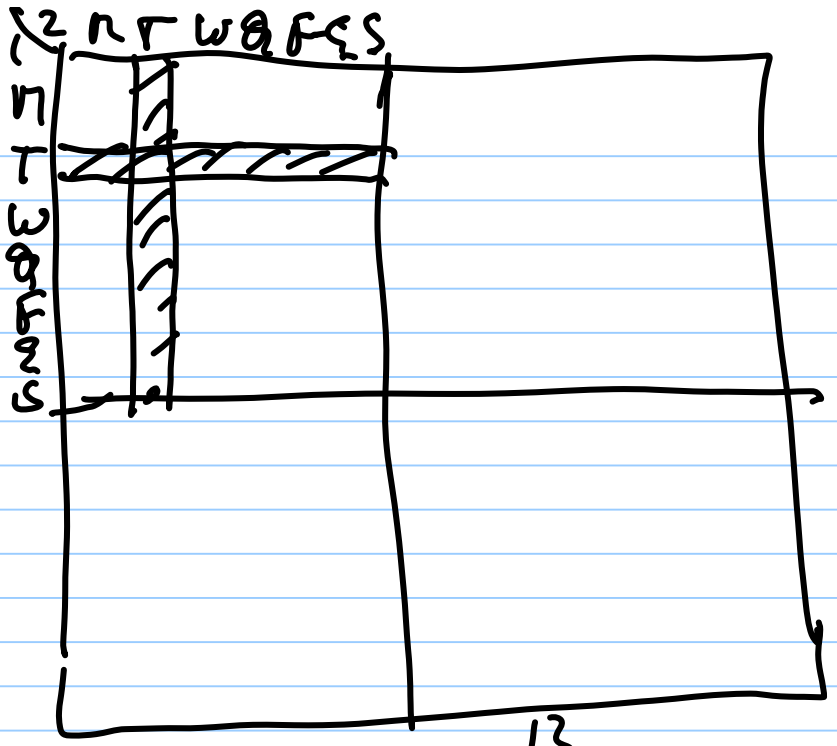
$$= \frac{27}{196}$$

E - both males



M F

$E \cap T$



$$P(E \cap T) = \frac{7+7-1}{14^2} = \frac{13}{196}$$

$$P(E \mid T) = \frac{P(E \cap T)}{P(T)} = \frac{\frac{13}{196}}{\frac{27}{196}} = \frac{13}{27}.$$

Note: Asking: Suppose a person has two children at least one is a male. What is the probability the other one is a male born on a Tuesday?

which one?

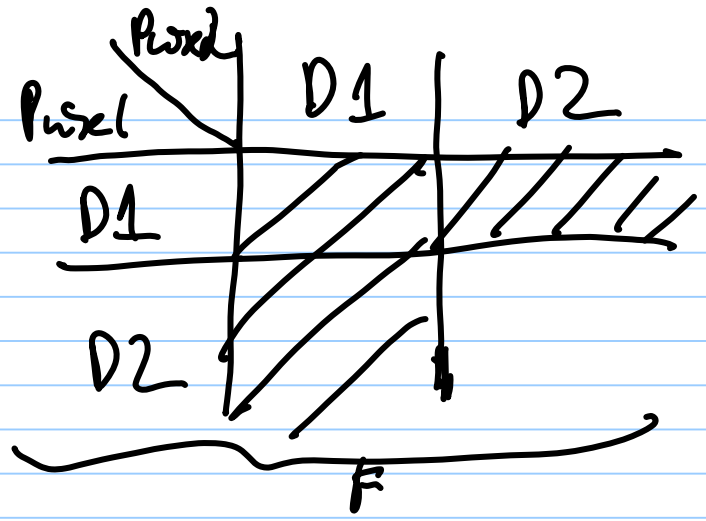
does not make sense. (is ambiguous)

Simplified Let's Make a Deal!

Game show: Two doors are on stage and

behind them, two prizes are randomly placed. The contestant points at one door, if there is at least one prize behind it, it is revealed and then removed (This is the practice round.) In the second round, the contestant again chooses a door. Again, if a prize is left behind that door, it is revealed and this time, the contestant gets to keep it. What is the right strategy?

Solution:



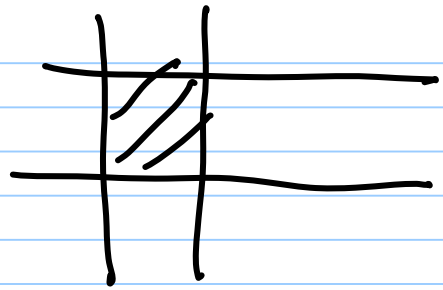
The first door should be chosen randomly, e.g. D1.

If no prize is revealed in 1st round, choose D2 - certain win

If there is a prize behind D1 conditioning event

$$P(F) = \frac{3}{4}.$$

E = both prizes behind Door 1: $E \subset F$



$$P(E \cap F) = P(E) = \frac{1}{4}$$

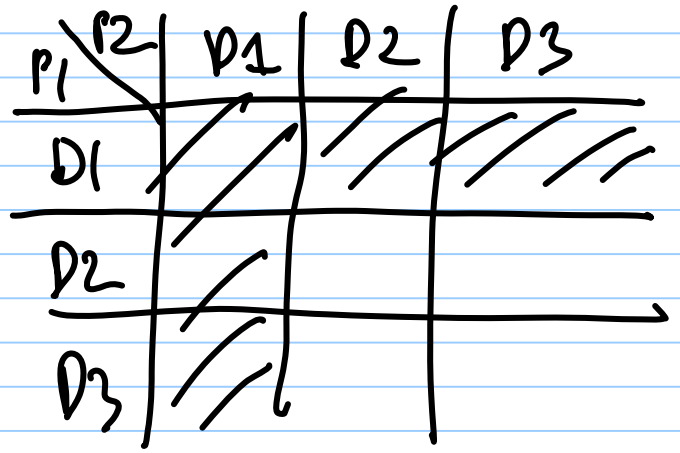
$$P(E|F) = \frac{1/4}{3/4} = \frac{1}{3}$$

Correct strategy: choose the other door in Round 2 no matter what

$$P(E^c|F) = 1 - P(E|F) = 1 - \frac{1}{3} = \frac{2}{3}$$

Follow-up: 3 doors.

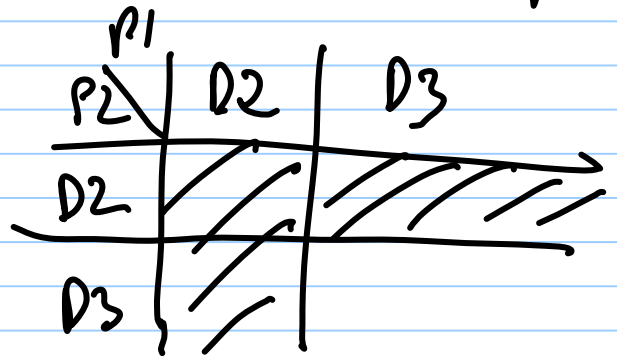
If there was a prize behind D1:



Conditioning event: $\bar{F}$

First and No prize
$\Rightarrow$ chose one of the other doors

Chance of winning: $\frac{3}{4}$



(B)

choose door 2

(there was a prize behind door 1)

$P(F) = \frac{5}{9}$ $\qquad E$

Probability that both prizes behind door 1 : $P(E) = \frac{1}{9}$

$E \subset F$ $\qquad P(E|F) = \frac{P(E)}{P(F)} = \frac{P(E \cap F)}{P(F)} = \frac{1/9}{5/9} = \frac{1}{5}$

$P(E^c) = \frac{4}{5}$ $\qquad$ Choose, say D2

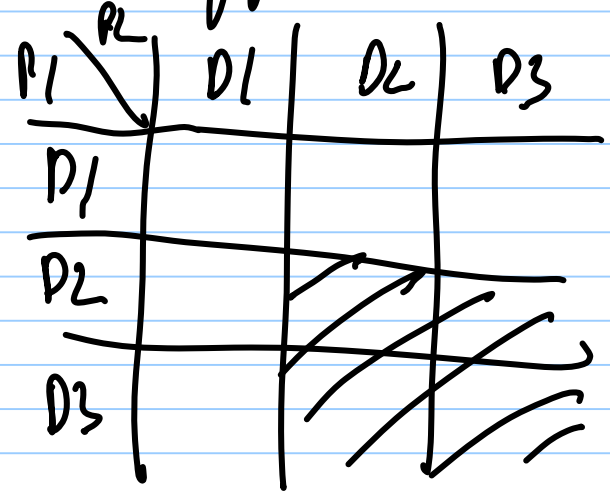
By symmetry: $P(\text{winning}) = \frac{2}{5}$.

Strategy: Choose another door (no matter which)

What are our chances of winning the game overall using this strategy?

$$P(G) = \frac{4}{9}$$

↑ no prize behind D1



$$\underbrace{\frac{4}{9}}_{P(G)} \cdot \frac{3}{4} \;\otimes\; \underbrace{\frac{5}{9}}_{P(G^c)} \cdot \frac{2}{5} = \frac{3}{9} + \frac{2}{9} = \frac{5}{9}$$

Bayes formula ← add up conditional probability over a complete system of disjoint conditions to get total probability.

HW ③ Suppose a person has two children and at least one of them is a male born on M or Tuesday. ⓐ What is the probability the other one is a female? ⓑ What is the probability the other one is a female born on a Thursday?

(4) Discuss "let's make a deal" with 4 doors. Assuming I choose another door in Round 2.

(a) What are my chances of winning if no prize was revealed in Round 1?

(b) What are my chances of winning if there was a prize revealed in Round 1?

(c) What are my overall chances of winning?