Note Title 9/30/2022

MATH 425 8/30/2022

Topics on Exam:

general formulas: Binomial theorem
Multinomial theorem
$1+a+a^2+\cdots$

partitions (together with different variants covered)

Sample spaces with equal likelihood of outcomes:
- committee problems (w/o replacement)
- with replacement

- Poker hands
- Poker dice
- Bridge distribution problems

More specific review on Monday

{- exclusion and inclusion principle

On with the material:

Bayes formula F conditioning event

$$P(E) = \underbrace{P(E|F)\cdot P(F)}_{P(E\cap F)} + \underbrace{P(E|F^c)\cdot P(F^c)}_{P(E\cap F^c)}$$

Example: A student who took AP BC has probability 0.8 of getting an A in Differential equations. The probability of getting an A in that course without the test is 0.5. If 20% of all students took the AP BC, what is the overall probability of getting an A in Diff. Eq.?

Solution: F = AP BC test $\quad P(F) = 0.2$

A = getting an A in Diff Eq

$P(A|F) = 0.8 \qquad P(A|F^c) = 0.5$

$$P(A) = P(A|F)\cdot P(F) + P(A|F^c)\cdot P(F^c) =$$

$$= 0.8 \cdot 0.2 + 0.5 \cdot 0.8 = \underline{\underline{0.56}}$$

<u>Follow-up question</u>: Suppose a student got an A in Diff. Eq. What is the probability they took the AP BC test?

Solution: $P(F|A) = \dfrac{P(F\cap A)}{P(A)} =$

$$= \frac{\overbrace{P(A|F)\cdot P(F)}^{0.16}}{\underbrace{P(A|F)\cdot P(F)}_{0.16} + \underbrace{P(A|F^c)\cdot P(F^c)}_{0.4}} =$$

reversed conditioning

$$= \frac{0.16}{0.56} = \underline{\underline{\frac{2}{7}}}.$$

Example: (Simplified insurance problem)

There is a psychological risk factor R which makes drivers more likely to have accidents.

A driver who has R has an accident during 1 year with probability 0.2.

A driver who does not have R has an accident during 1 year with probability 0.1.

30 % of all drivers have R.

(a) What is the probability a driver who did have an accident in the first year of insurance has R?

(b) What is the probability a driver who did not have an accident in the first year has R?

<u>Solution:</u> R = conditioning event

A = accident during the first year

$P(R) = 0.3$ $\quad P(A|R) = 0.2$ $\quad P(A|R^c) = 0.1$

$P(R) + P(R^c) = 1$

$$P(A) = \underbrace{\underset{0.2}{P(A|R)} \cdot \underset{0.3}{P(R)}}_{0.06} + \underbrace{\underset{0.1}{P(A|R^c)} \cdot \underset{0.7}{P(R^c)}}_{0.07}$$

$$= 0.13$$

$$P(R|A) = \frac{\overbrace{P(A|R) \cdot P(R)}^{0.06}}{\underbrace{P(A)}_{0.13}} = \underline{\underline{\frac{6}{13}}} \quad a$$

$$P(A^c) = P(A^c|R) \cdot P(R) + P(A^c/R^c) \cdot P(R^c)$$

$$0.8 \cdot 0.3 \qquad 0.9 \cdot 0.7$$

$\underbrace{\qquad}_{0.24}$ $\underbrace{\qquad}_{0.63}$

$$= 0.87$$

$$\boxed{\text{check: } P(A)+P(A^c) = 1}$$

$$P(R|A^c) = \frac{P(A^c|R)\cdot P(R)}{P(A^c)} = \frac{0.24}{0.87} = \frac{8}{29}$$

$$P(R|A) + P(R|A^c) \neq P(R)$$

$$\frac{P(R\cap A)}{P(A)} + \frac{P(R\cap A^c)}{P(A^c)}$$

$$P(R\cap A) + P(R\cap A^c) = P(R)$$

HW 5 The probability that I will get an A on a history test if I studied is 0.7.
The probability I get an A if I did not study is 0.3. If 40% of all students studied for the test, what percentage of students will have an A?

6 In the setup of problem 5,

what percentage of students who got an A studied?