MATH 425 10/3/2022

Note Title 10/3/2022

Review

Formulas: $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$

$$\binom{n}{k} = \binom{n}{n-k}$$

Example: $\binom{20}{0} - 2\binom{20}{1} + \cdots + 2^{20}\binom{20}{20} =$

$a = 2$

$b = -1$

$= (2-1)^{20} = \underline{\underline{1}}$

$$1 + a + a^2 + \cdots = \frac{1}{1-a} \qquad -1 < a < 1$$

Example: $\frac{1}{5} + \frac{1}{5^2} + \cdots = \frac{1}{5}\left(1 + \frac{1}{5} + \frac{1}{5^2} + \cdots\right)$

$$a = \frac{1}{5} \qquad = \frac{1}{5}\left(\frac{1}{1-\frac{1}{5}}\right) = \frac{1}{4}$$

$$2^{-1}\, 2^{-1/4}\, 2^{-1/4^2} \cdots = 2^{-1 - \frac{1}{4} - \frac{1}{4^2} - \cdots} \qquad -\cdots = \frac{1}{1-1/4}$$

$$= \boxed{2^{-4/3}}$$

	number of choices (by number of each kind)	
Partitions: choose n pieces of candy, k different kinds. $n = 10$, $k = 4$ } **Example**	$\binom{n+k-1}{k-1}$	$\binom{13}{3}$
variant 1: Each kind must be chosen at least once	$\binom{n-1}{k-1}$	$\binom{9}{3}$
Variant 2: choose at most n pieces of candy	$\binom{n+k}{k}$	$\binom{14}{4}$

Without or with replacement

Committee problem:

20 men 10 women

committee of 8 people selected at random.

What is the probability that 5 are men and 3 are women?

$$\boxed{\frac{\binom{20}{5}\binom{10}{3}}{\binom{30}{8}}}$$

Without replacement

20 men 10 women

8 times in a row, people are selected at random to answer questions (can be selected again)

Probability that 5 men and 3 women will have answered questions

With replacement

ordered sample spaces

$$\boxed{\binom{8}{5}\left(\frac{2}{3}\right)^5 \cdot \left(\frac{1}{3}\right)^3}$$

$$p\left(\text{man selected for one question}\right) = \frac{20}{20+10} = \frac{2}{3}$$

Poker hands (without replacement) $|S| = \binom{52}{5}$

3 of a kind $aaabc$

denomination

$$\boxed{\frac{13\cdot\binom{4}{3}\cdot\binom{12}{2}\cdot 4^{2}}{\binom{52}{5}}}$$

2 pair $\overbrace{aa\,bb}^{\binom{13}{2}}\,\overset{11}{c}$

$$\boxed{\frac{\binom{13}{2}\cdot\binom{4}{2}^{2}\cdot 11\cdot 4}{\binom{52}{5}}}$$

2 <u>pair</u> pairs are Aces and Kings

AAKKc

$$\boxed{\frac{\overset{A}{\binom{4}{2}}\ \overset{K}{\binom{4}{2}} \cdot 11 \cdot 4}{\binom{52}{5}}}$$

<u>Full House</u> aaabb

$$\boxed{\frac{13 \cdot 12 \cdot \binom{4}{3}\binom{4}{2}}{\binom{52}{5}}}$$

Flush

5 cards
in the same suit
not a sequence

$$\boxed{\dfrac{4\cdot\left(\binom{13}{5}-10\right)}{\binom{52}{5}}}$$

not a straight flush

⋮

Poker dice

with replacement

$|S| = 6^5$

5 standard cubic dice

2 pair aabbc

pairs not distinguishable

$$\boxed{\frac{\frac{1}{2}\binom{5}{2\ 2\ 1}\cdot 6\cdot 5\cdot 4}{6^5}}$$

4 dividers with 2 pairs

1 2 3 4

(12)(34) (13)(24)

(K+) [23] ← 3 choices

$\frac{1}{2}\binom{4}{2}$

P(two pair including a pair of 6s)

66 bb c

$$\frac{\binom{5}{2} \cdot 3 \cdot 5 \cdot 4}{6^5}$$

where is c

Bridge $\stackrel{?}{=}$ P some (unspecified) player has exactly 7 spades?

which player (impossible for two to have 7 spades)

$$\boxed{4 \cdot \frac{\binom{13}{7}\binom{39}{6}}{\binom{52}{13}}}$$

specific player has exactly 7 spades

P (N & S have exactly 7 spades together)

$$\frac{\binom{13}{7}\binom{39}{\underbrace{19}_{26-7}}}{\binom{52}{26}}$$

$|S| = \binom{52}{26}$

combined hand of N and S

What is the probability that one partnership (NS or EW) have all 4 aces?

unspecified which partnership

$$\boxed{2 \cdot \frac{\binom{48}{22}}{\binom{52}{26}}}$$

52 − 4

26 − 4

A football team has 16 offensive and 16 defensive players. Housed in a hotel in rooms of 2. What is the probability that all offensive players will be in a room with an offensive player?

$$\frac{\binom{16}{2\ 2\ 2\ \cdots\ 2}}{8!} \cdot \frac{\binom{16}{2\ 2\ 2}}{8!}$$

$$\frac{\qquad}{\underbrace{\binom{32}{2 \qquad 2}}_{16!}} = \boxed{\frac{16!^3}{8!^2\, 32!}}$$

What is the probability that offensive players will be in rooms 1-8?

rooms 9-16

$\underbrace{\text{16 beds}}_{2}$

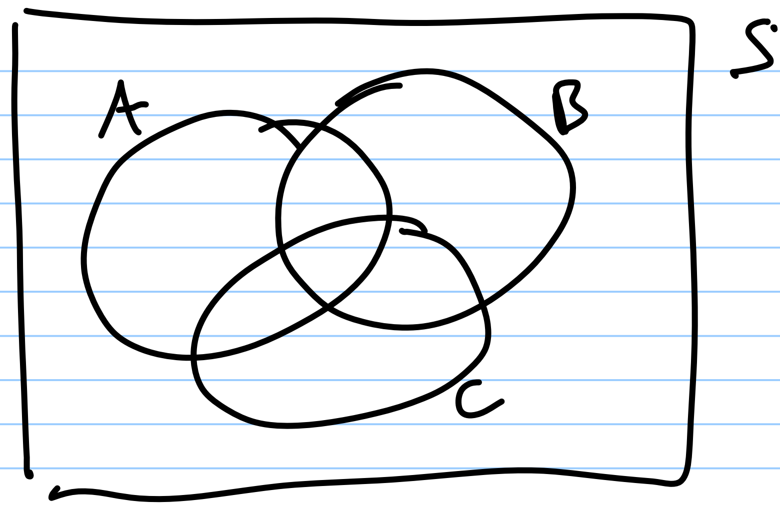
16 beds

1 ← all offensive players

$\binom{32}{16}$

players on rooms 1-8

Inclusion and exclusion principle:



$|S| = 80$

$|A| = 25 \quad |B| = 30$

$|C| = 40$

$|A \cap B| = 15 \quad |A \cap C| = 25$

$|B \cap C| = 10$

$|A \cap B \cap C| = 5$

? $|A \cup B \cup C| = \cancel{25} + 30 + 40 - 15 - \cancel{25} - 10 + 5$

$= 70 - 25 + 5 = \underline{\underline{50}}$

$$P((A \cup B \cup C)^c) = \frac{30}{80} = \frac{3}{8}$$

$$|A \cup B \cup C|^c = 80 - 50 = 30$$