MATH 425 10/7/2022

Example: On a multiple-choice test (no penalty for wrong answers) the student knows the answer or guesses randomly. There are m choices. The probability of knowing the answer is p. What is the probability that a student who got a correct answer knew it?

Solution: K = knew the answer C = got the correct answer

$$P(K) = p \qquad P(C|K) = 1 \qquad P(C|K^c) = \frac{1}{m}$$

$$P(K|C) = \frac{\overset{1}{P(C|K)}\,\overset{p}{P(K)}}{\underbrace{P(C|K)}_{1}\cdot\underbrace{P(K)}_{p} + \underbrace{P(C|K^c)}_{1/m}\cdot\underbrace{(1-P(K))}_{(1-p)}} = \frac{p}{p + \frac{(1-p)}{m}} = \frac{mp}{1+(m-1)p}$$

$$\frac{mp}{1+(m-1)p}$$

e.g. 5 choices, 60% know

$m=5 \quad p=0.6$

$$\frac{3}{1+2.4} = \underline{\underline{\frac{3}{3.4}}}$$

5 choices, 30% know

$$\frac{1.5}{1+1.2} = \underline{\underline{\frac{1.5}{2.7}}}$$

Example: A blood test is 95% effective in detecting a disease (5% false negative) Probability of a false positive is 1%. 0.5% of the population has the disease. If a patient's test comes back positive, what is the probability they have the disease?

Solution: D = have the disease $\quad P(D) = 0.005$

P = tested positive $\quad P(P|D) = 0.95 \quad P(P|D^c) = 0.01$

$$P(D|P) = \frac{P(P|D)\,P(D)}{P(P|D)\,P(D) + P(P|D^c)(1 - P(D))} = \frac{0.95 \cdot 0.005}{0.95 \cdot 0.005 + \underline{0.01 \cdot 0.995}}$$

$\approx \underline{\underline{0.323}}$ (in the book)

Probability as a measure of belief (we depart from our mathematical setup)

Example: A police investigator is 60% sure the suspect is guilty. A new piece of evidence shows the perpetrator was left-handed. 20% of the population is left-handed. If the suspect is also left-handed, how certain should the investigator be of their guilt?

Solution: G = guilty

L = left-handed

$$P(G|L) = \frac{\overbrace{P(L|G)}^{1} \cdot \overbrace{P(G)}^{0.6}}{\underbrace{P(L|G)}_{1} \cdot \underbrace{P(G)}_{0.6} + \underbrace{P(L|G^c)}_{0.2} \cdot \underbrace{(1-P(G))}_{0.4}} = \frac{0.6}{0.68} \approx \underline{\underline{0.882}}$$

(in the book)

<u>Odds</u>. If an event A has probability P(A). The <u>odds</u> are

$$\boxed{O(A) = \frac{P(A)}{1 - P(A)}}$$

$P \in [0,1]$, $O(A) \in [0,\infty]$.

<u>Example</u>: The probability of Michigan football team winning a game at Northwestern is 0.8. What are the odds of Michigan winning?

<u>Solution</u>: $\frac{0.8}{0.2} = \underline{\underline{4}}$ $4:1$

Significance of odds in betting: If I am running the book (just for an honor bet - to break even), how much should I pay out on a dollar 1\$ bet on Michigan to win?

$$1 + x$$

return the bet; winnings

Michigan wins: $-x$
Michigan loses: 1
winnings of the book

$$\underbrace{\frac{4}{5}\cdot(-x) + \frac{1}{5}\cdot 1 = 0}_{x = \frac{1}{4}}$$

P(Michigan wins)

$$\frac{1}{\text{odds}}$$

How much can I offer on a \$1 bet for Northwestern to win?

$$\boxed{x = 4}$$

HW ① In a store, there are two types of apples, F and G. F has probability 0.05 to be rotten, G has probability 0.1 to be rotten. 20% of the apples are of type F, (80% of type G). If I pick a random apple and it is rotten, what is the probability it is of type G?

(2) In the setup of the previous problem, if the apple is not rotten, what is the probability it is of type F?

(3) A basketball team T will win a game with probability 0.95. How much in winnings should I pay out on a \$1 bet

(a) for T to win

(b) for T to lose

to break even?