MATH 425 10/10/2022

Calculating probability from odds:

$$O(E) = \frac{P(E)}{1-P(E)}$$

solve for $P(E)$:

$$O(E) - O(E)P(E) = P(E)$$

$$\boxed{P(E) = \frac{O(E)}{1+O(E)}}$$

Example: If the odds of winning a game are 5:2, what is the probability of winning it?

solution: $O(E) = \frac{5}{2}$

$$P(E) = \frac{O(E)}{1+O(E)} = \frac{\frac{5}{2}}{\frac{7}{2}} = \frac{5}{7}$$

Remaining topics of basic probability

Conditioning on more than two possibilities

Example: A bin contains 3 types of flashlights. Probabilities that a flashlight lasts > 100 hours, and frequencies of each type:

type	$P(>100)$	frequency
1	0.7	20%
2	0.4	30%
3	0.3	50%

ⓐ What is the probability that a random flashlight lasts > 100 h?

ⓑ If a flashlight lasted > 100 h, what is the probability it is of type 1, 2, 3?

Solution :

	$P(>100)$	frequency
1	0.7	20%
2	0.4	30%
3	0.3	50%

$$P(>100) = \overset{0.7}{P(>100/T1)} \cdot \overset{0.2}{P(T1)} + \overset{0.4}{P(>100/T2)} \cdot \overset{0.3}{P(T2)}$$

$$+ \overset{0.3}{P(>100/T3)} \cdot \overset{0.5}{P(T3)} = \underline{\underline{0.41}} \quad (a)$$

For Question (b), just choose the appropriate summand & divide by total probability.

$$P(T1 | >100) = \frac{P(>100 | T1) \cdot P(T1)}{P(>100)} = \frac{0.14}{0.41} = \underline{\underline{\frac{14}{41}}}$$

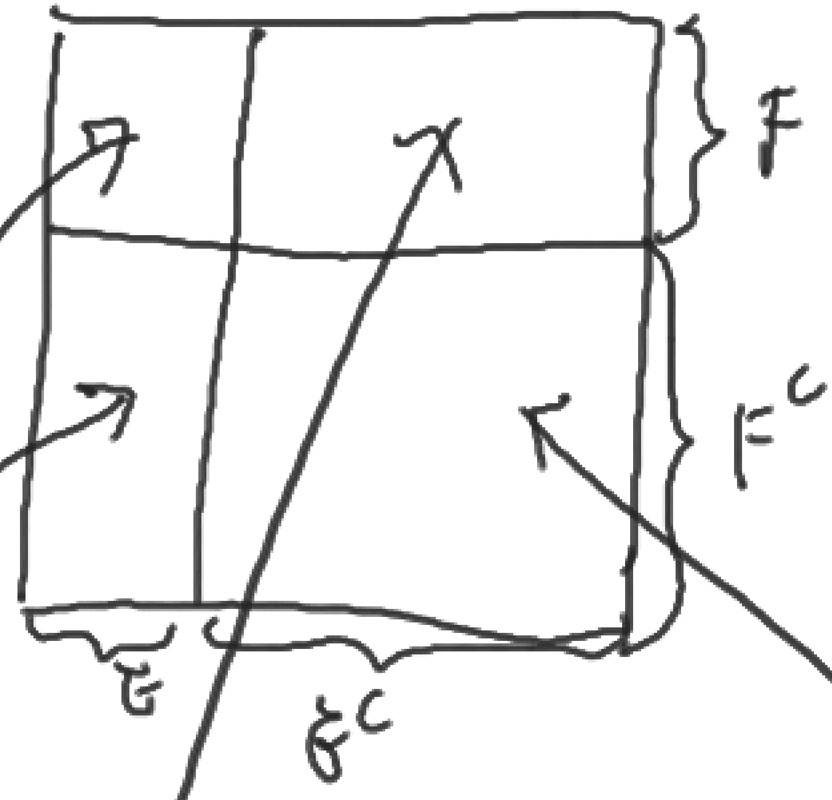
$$P(T2 | >100) = \frac{0.12}{0.41} = \underline{\underline{\frac{12}{41}}} \qquad P(T3 | >100) = \frac{0.15}{0.41} = \underline{\underline{\frac{15}{41}}}$$

Independent Events

Two events E, F are *independent* when $P(E \cap F) = P(E)P(F)$. (Equivalently, $P(E) = P(E|F)$
Equivalently, $P(F) = P(F|E)$.)

Venn diagram:

independent events in this picture mean that probability is proportional to area:



$P(E \cap F) = P(E)P(F)$ $\therefore$ $P(F \cap E^c) = P(F)(1 - P(E))$

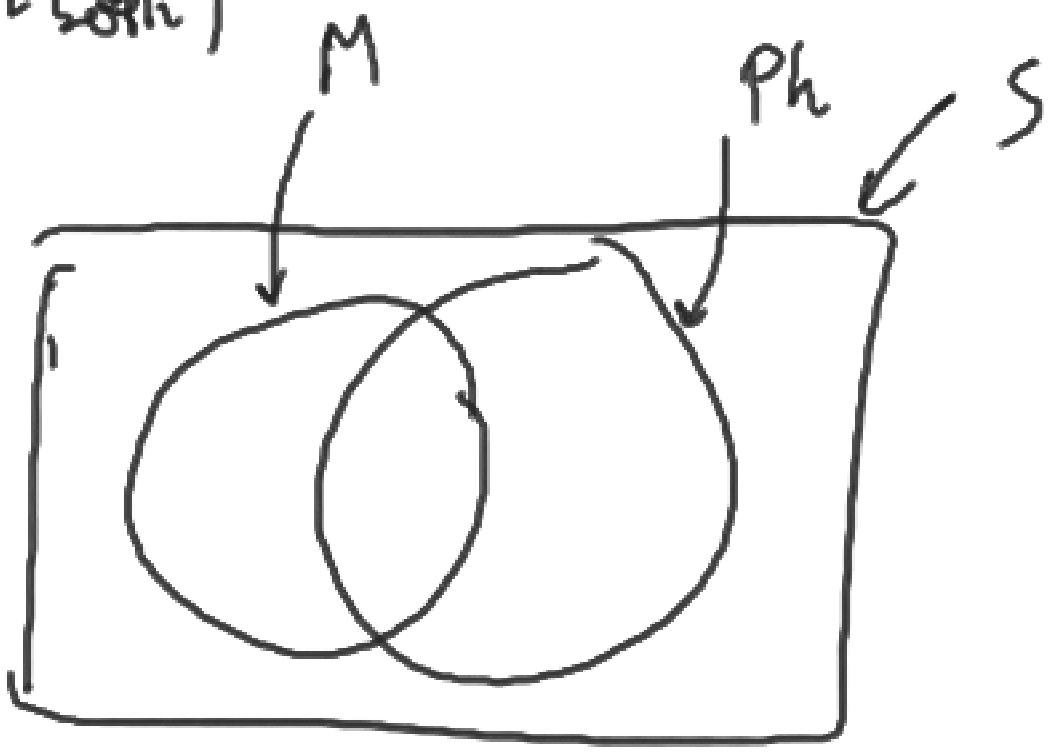
$P(E^c \cap F^c) = (1 - P(E))(1 - P(F))$

$P(E \cap F^c) = P(E)(1 - P(F))$

Example: In a class of 16 students, 8 take math, 4 take physics 10 take math or physics. Are the events of taking math and physics independent?

(or both)

Solution:



$$|S| = 16$$
$$|M| = 8$$
$$|Ph| = 4$$
$$|M \cup Ph| = 10$$

$$P(M) = \frac{8}{16} = \frac{1}{2}$$
$$P(Ph) = \frac{4}{16} = \frac{1}{4}$$
$$P(M \cup Ph) = \frac{10}{16} = \frac{5}{8}$$

$$P(M \cap Ph) = P(M) + P(Ph) - P(M \cup Ph) =$$
$$= \frac{1}{2} + \frac{1}{4} - \frac{5}{8} = \frac{3}{4} - \frac{5}{8} = \frac{1}{8}$$

$$\frac{1}{8} = \frac{1}{2} \cdot \frac{1}{4}$$

Yes, taking Math and Physics are independent events in that class.

Follow up: Same scenario: 8 take math 4 take physics
10 take at least one of those
in a class of 32 students. Are taking Math and Physics independent events?

Solution: $|M| = 8 \quad |Ph| = 4 \quad |M \cup Ph| = 10 \quad |M \cap Ph| = 2$

In our new sample space $|S| = 32$

$$P(M) = \frac{1}{4}, \; P(Ph) = \frac{1}{8}, \; P(M \cap Ph) = \frac{1}{16}$$

$$\frac{1}{4} \cdot \frac{1}{8} < \frac{1}{16}$$ In the new sample space, the events are not independent

$\boxed{\text{SO CONDITIONING AFFECTS INDEPENDENCE OF EVENTS}}$!

Risk in consecutive years

Recall the insurance problem: A driver with a known risk factor R has probability 0.2 of having an accident in one year, without R the probability of an accident is 0.1. 30% of all drivers have R.

Suppose the driver does have an accident in Year 1 of their coverage. What is the probability they will have an accident in Year 2?

Assumption: independent events for a given driver

Solution:

R = Risk factor

A_1 = accident in Year 1

A_2 = accident in Year 2

$P(R) = 0.3$

$P(A_1 | R) = 0.2$

$P(A_1 | R^c) = 0.1$

$$P(A_1) = \underbrace{P(A_1|R)}_{0.2} \cdot \underbrace{P(R)}_{0.3} + \underbrace{P(A_1|R^c)}_{0.1} \cdot \underbrace{P(R^c)}_{0.7} = 0.13$$

$$P(A_1) = \underset{0.2}{P(A_1|R)} \cdot \underset{0.3}{P(R)} + \underset{0.1}{P(A_1|R^c)} \cdot \underset{0.7}{P(R^c)} = \underline{\underline{0.13}}$$

$$P(R|A_1) = \frac{0.2 \cdot 0.3}{0.13} = \underline{\underline{\frac{6}{13}}}$$

$$P(A_2|A_1) = P(A_2|R\cancel{A_1}) \cdot P(R|A_1) + P(A_2|R^c\cancel{A_1}) \cdot P(R^c|A_1)$$

$$= \underset{0.2}{P(A_2|R)} \cdot \underset{\frac{6}{13}}{P(R|A_1)} + \underset{0.1}{P(A_2|R^c)} \cdot \underset{\frac{7}{13}}{P(R^c|A_1)} = \frac{1.9}{13} = \underline{\underline{\frac{19}{130}}}$$

Adjust conditioning probabilities based on new information

HW ① Suppose the odds of winning a bet are 1 : 1.25.
What is the probability of winning it?

② In a grocery store, there are 3 types of apples: F, G, H. Table of likelihood of being rotten, and frequency of each type:

type	prob. of rotten	frequency
F	0.1	40%
G	0.05	20%
H	0.2	40%

Given that I picked an apple that is rotten, what are the probabilities it was of type F, G, H?

(3) Suppose 40% of people have brown hair, 5% own a red car. Suppose 42% either have brown hair or own a red car. Are having brown hair and owning a red car independent events?

(4) Suppose patients with a certain gene have probability 20% of getting a certain disease in a span of 5 years and people who do not have the gene have probability 1% of getting the disease in 5 years. (Assume in each group, getting it in the next 5 years is an independent event from the first 5 years.)

Suppose 5% of all people have the gene.

If an insurance client did not get the disease during the first 5 years of coverage, what is the probability they will get it in the next 5 years of coverage?