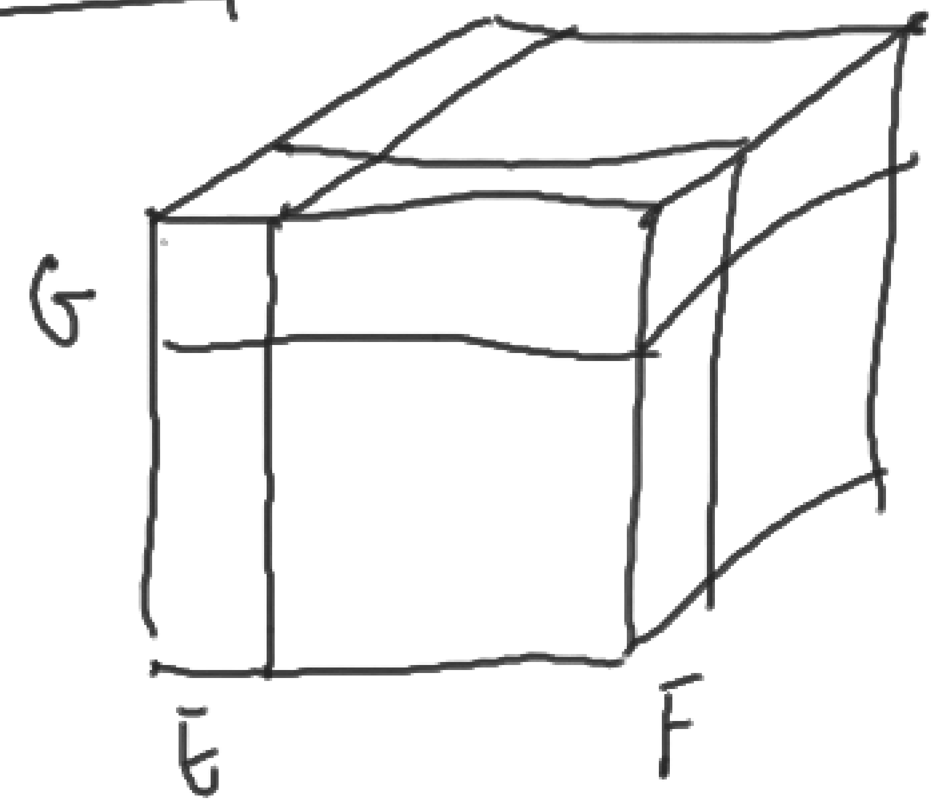
MATH 425

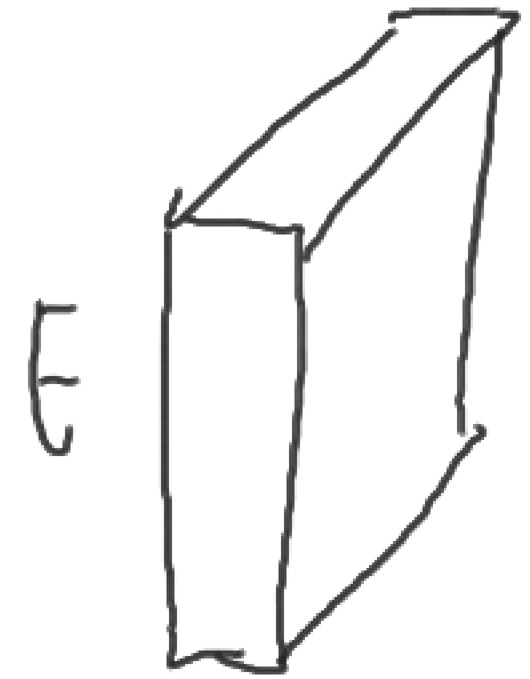
10/12/2022

Independence of more than 2 events:

(think of the volume model)

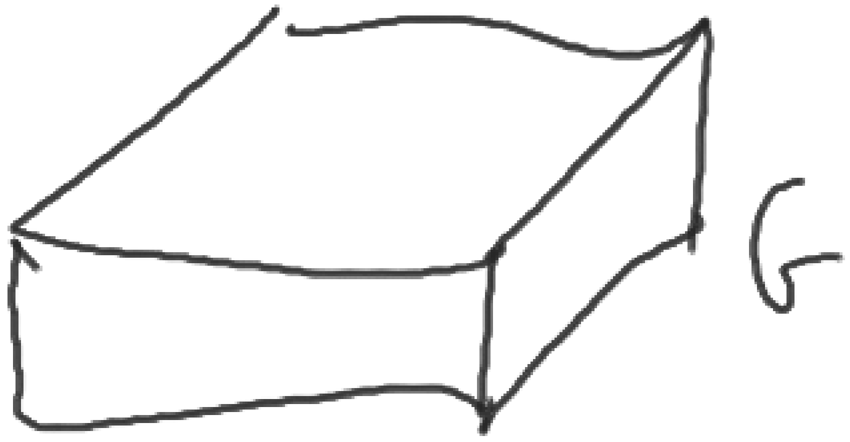
The case of 3 events





Venn Diagram

For independent events, the probability of all intersections is proportional to volume



$$P(E \cap F) = P(E)P(F)$$

$$P(E \cap G) = P(E)P(G)$$

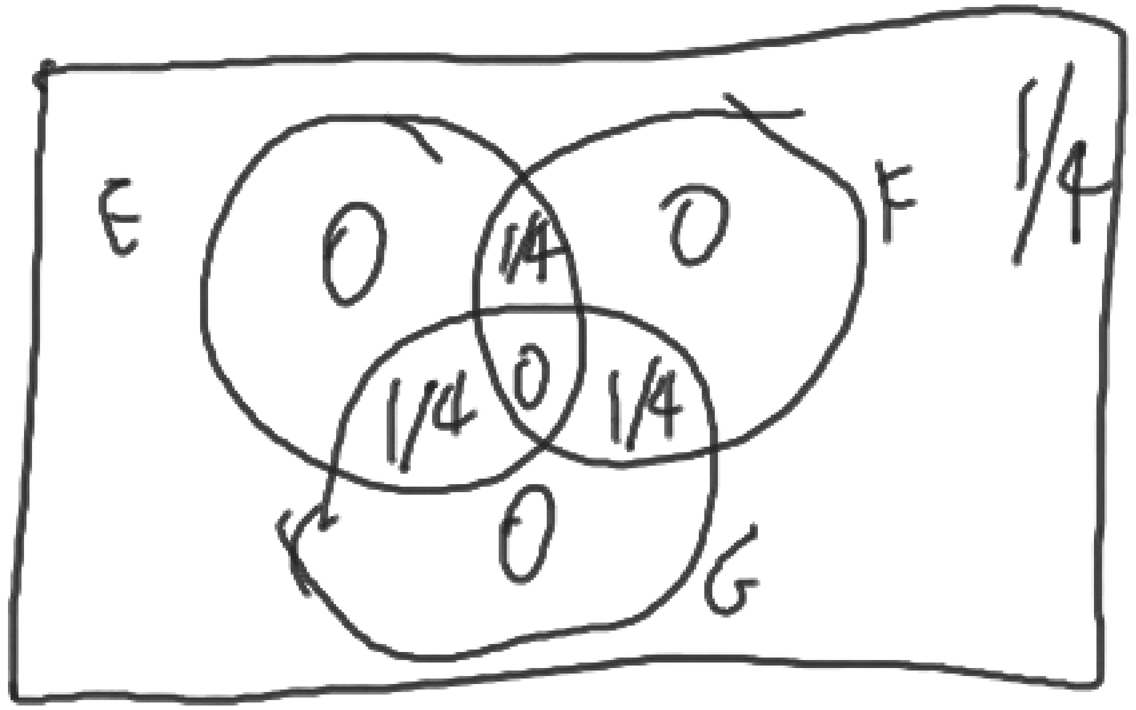
$$P(F \cap G) = P(F)P(G)$$

$$P(E \cap F \cap G) = P(E)P(F)P(G)$$

This condition does not follow from the others!

<u>Example</u>: Give an example of three events E, F, G where any two are independent but all three are not independent.

<u>Solution</u>:



$$P(E) = P(F) = P(G) = \frac{1}{2}$$

$$P(E \cap F) = P(E \cap G) = P(F \cap G) = \frac{1}{4}$$

Note: If E, F, G are (jointly) independent then the product ("volume") formula can be applied also to complements: For example,

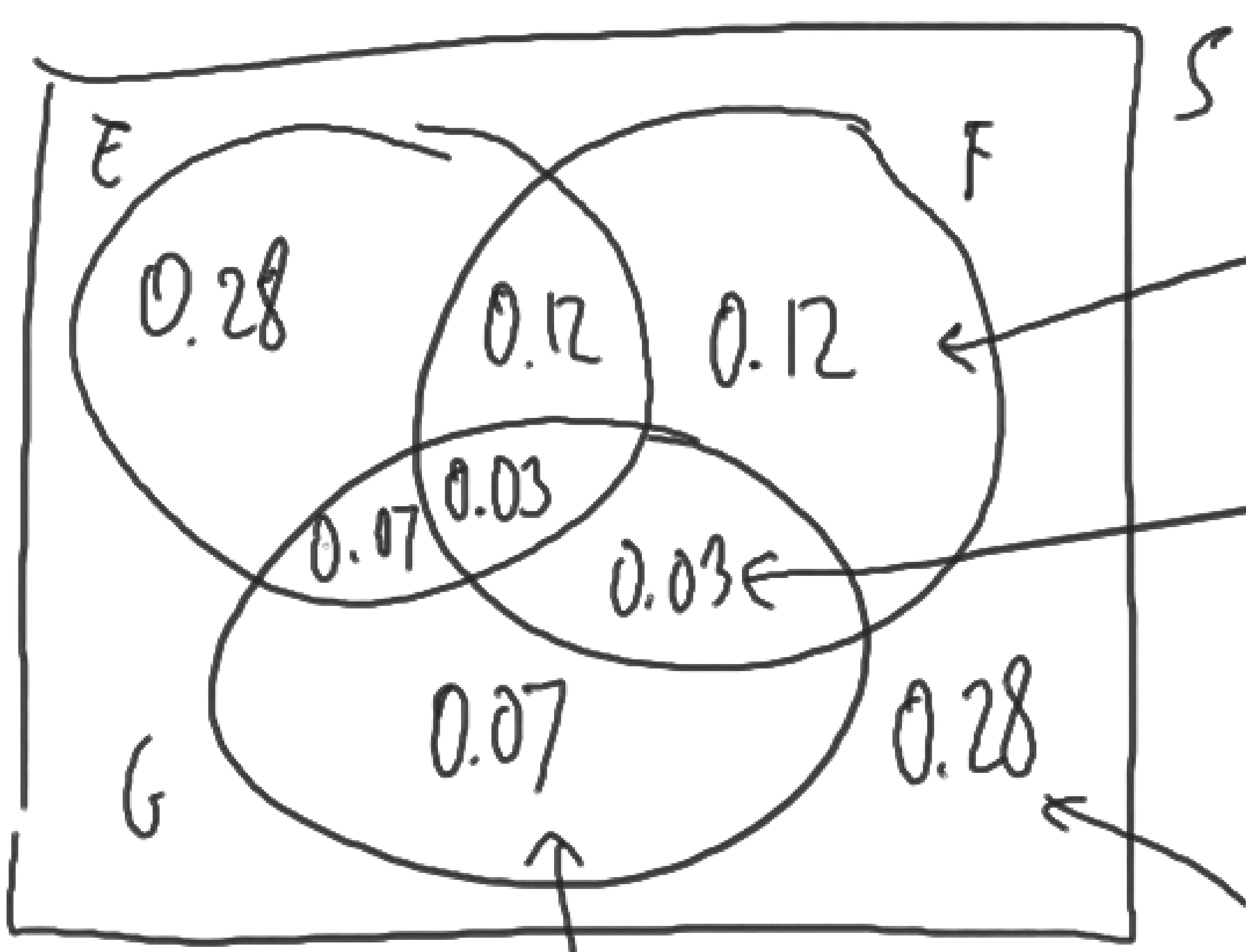
$$P(E^c \cap F \cap G^c) = P(E^c)P(F)P(G^c) =$$
$$= (1-P(E))P(F)(1-P(G)).$$

For more than 3 events $E_1, \dots, E_n$, independence means that the product formula holds for every subset of them: For $1 \le i_1 < \dots < i_k \le n$

$$P(E_{i_1} \cap \dots \cap E_{i_k}) = P(E_{i_1}) \cdot \dots \cdot P(E_{i_k}).$$

Example: Draw a Venn diagram with the probabilities of all intersections (and their complements) between three independent events E, F, G where $P(E) = 0.5$, $P(F) = 0.3$, $P(G) = 0.2$.

Use $P(\bar{E}) = P(E^c)$



$F \cap E^c \cap G^c$

$0.3 \cdot 0.5 \cdot 0.8$

$E^c \cap F \cap G$

$0.5 \cdot 0.3 \cdot 0.2$

$0.5 \cdot 0.7 \cdot 0.8$

$E^c \cap F^c \cap G^c$

$E^c \cap F^c \cap G$ $\quad 0.5 \cdot 0.7 \cdot 0.2$

$P(E^c) = 0.5$

$P(F^c) = 0.7$

$P(G^c) = 0.8$

Reliability of networks (basic cases)

Example: A communication link is *serial*; It has 3 nodes S1, S2, S3 (independent)

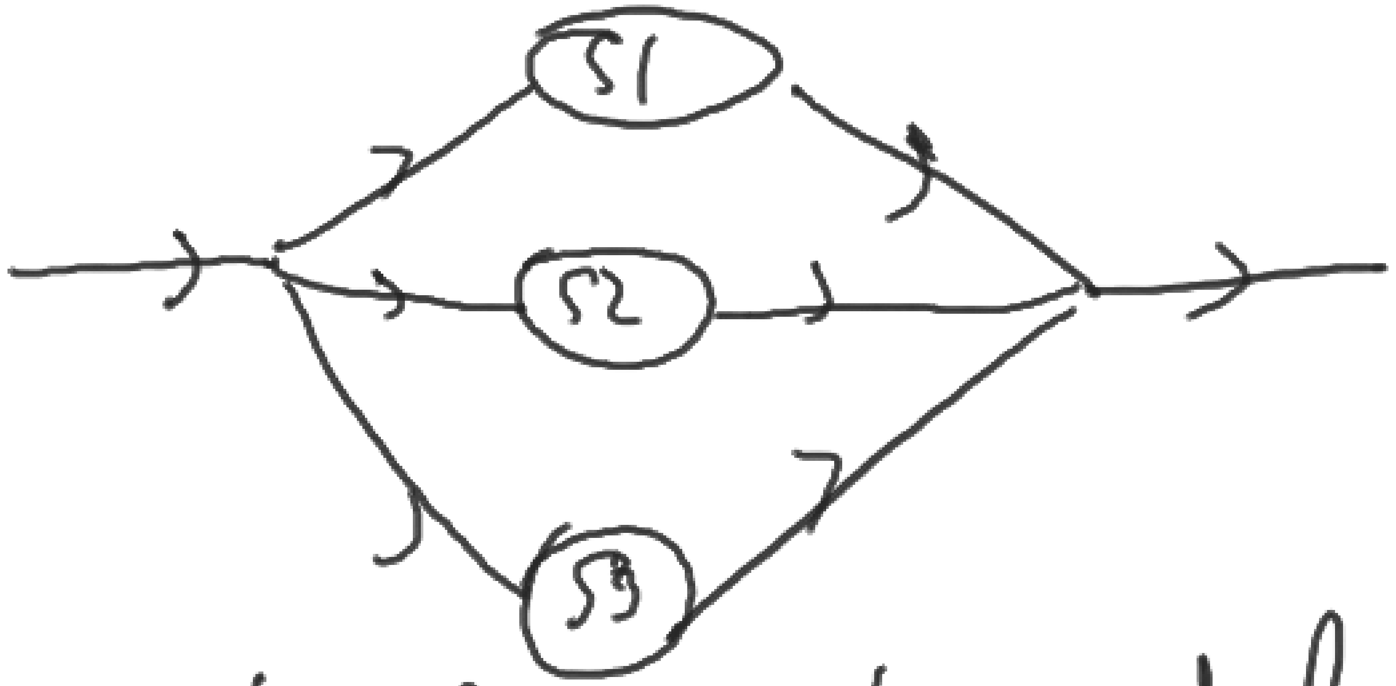
All three must work for the link to function. Probability of *failure* of S1, S2, S3 on a given day are 0.1, 0.2, 0.3. What is the probability that the link will work?

$$\begin{array}{r} 0.56 \\ 9 \\ \hline 0.504 \end{array}$$

Solution: S1, S2, S3 = these links work $P(S1^c) = 0.1$, $P(S2^c) = 0.2$, $P(S3^c) = 0.3$

$$P(S1 \cap S2 \cap S3) = P(S1)\,P(S2)\,P(S3) = 0.9 \cdot 0.8 \cdot 0.7 = \underline{0.504}$$

Example:

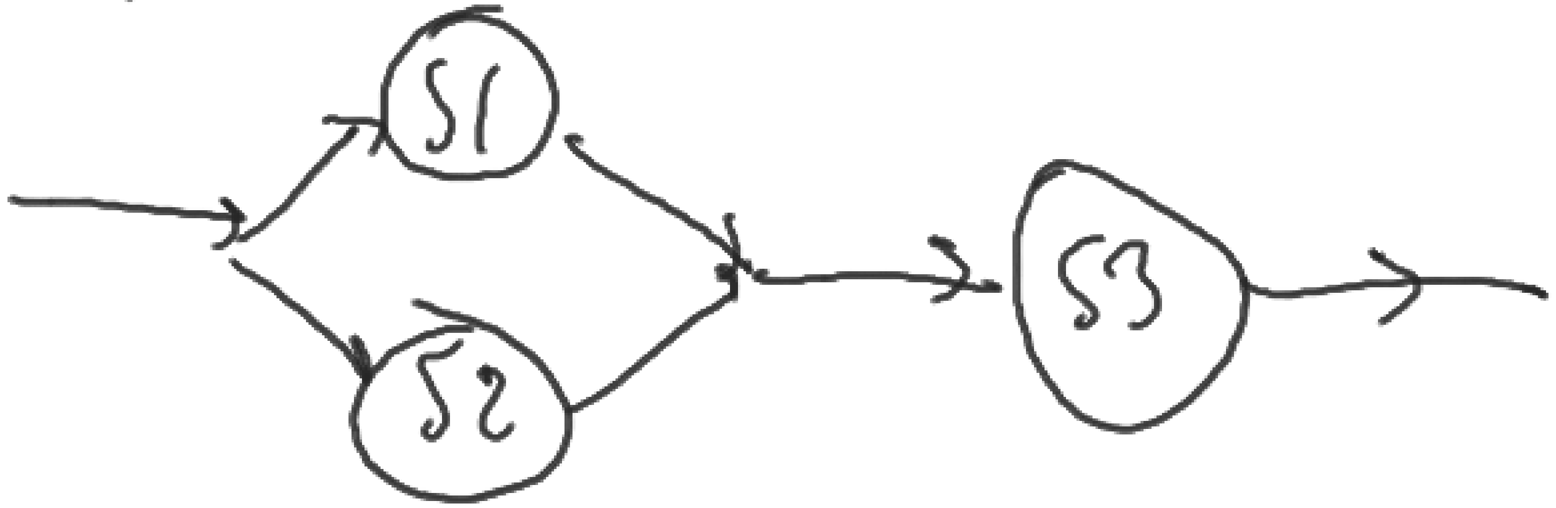


This communication link works if at least one of the nodes S1, S2, S3 work. Their failure probabilities are the same: 0.1, 0.2, 0.3. What is the probability this link will work?

Solution: $P(\underbrace{S1^c \cap S2^c \cap S3^c}_{\text{the link will fail}}) = 0.1 \cdot 0.2 \cdot 0.3 = 0.006$

$$P(S1 \cup S2 \cup S3) = 1 - 0.006 = \underline{\underline{0.994}}$$

Example: What about the reliability of this link:



$P(S1^c) = 0.1$, $P(S2^c) = 0.2$, $P(S3^c) = 0.3$

Solution: $P((S1 \cup S2) \cap S3) =$

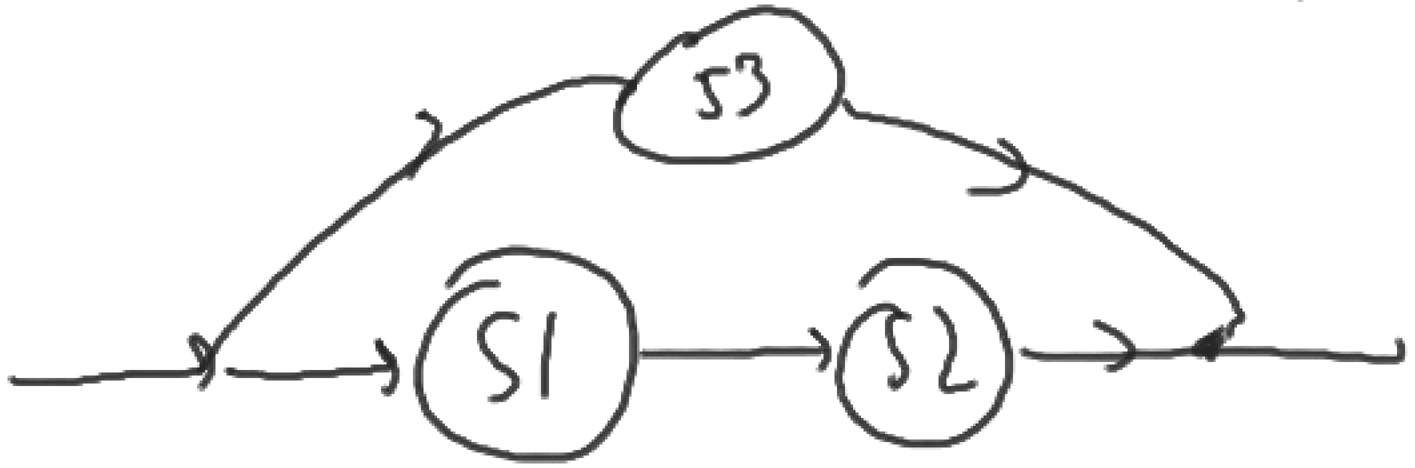
also independent events

$P(S1 \cup S2) =$

$1 - P(S1^c)\,P(S2^c) = 1 - 0.02 = 0.98$

$0.98 \cdot 0.7 = \underline{\underline{0.686}}$

Example : What is the reliability of the link



Either S3 works or both S1 & S2 must work (or both)

$(S1 \cap S2) \cup S3$

$P(S1^c) = 0.1, \; P(S2^c) = 0.2, \; P(S3^c) = 0.3$

$P(S1 \cap S2) = P(S1)P(S2) = 0.9 \cdot 0.8 = 0.72$

$P((S1 \cap S2) \cup S3) = 1 - P((S1 \cap S2)^c \cap S3^c) = 1 - 0.28 \cdot 0.3 =$

$1 - P(S1 \cap S2)^c \cdot P(S3^c) = 1 - 0.084 = \underline{\underline{0.916}}$

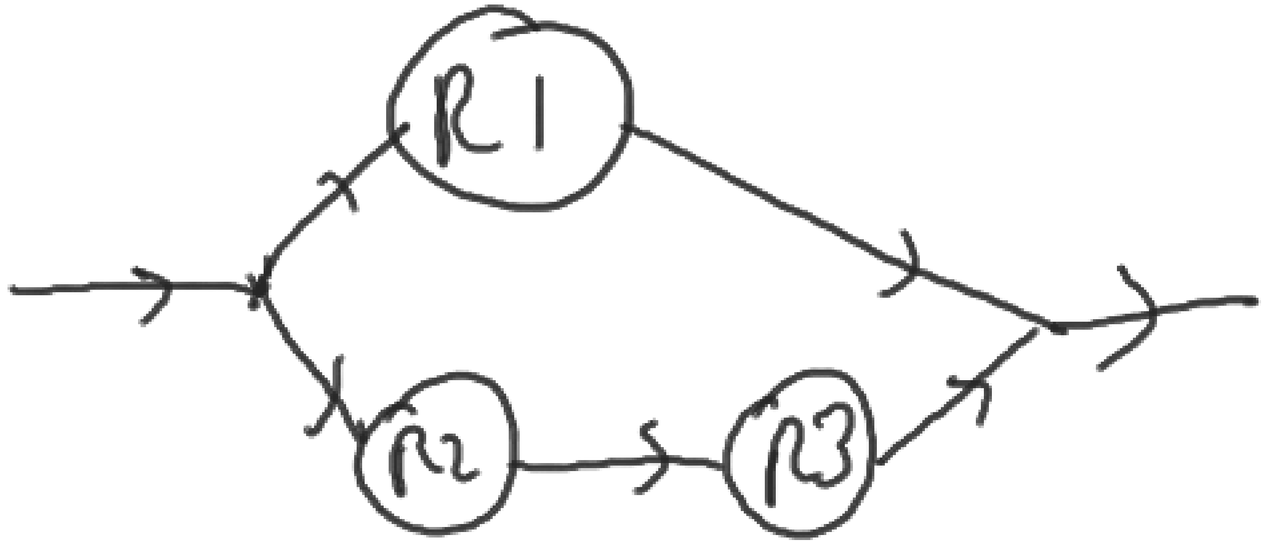
HW 5 : In a class of 100 students, 50 take Calculus, 40 take History and 10 take Music. If taking Calculus, History, and Music are independent events, draw a complete Venn diagram displaying all probabilities of taking or not taking each combination of these classes.

6 3 relays R1, R2, R3 are in a circuit. Probabilities that R1, R2, R3 work are R1: 0.8

R2 0.7

R3 0.6

What is the probability that the circuit will work in the following configuration:



(7) Same question for the configuration