MATH 425 10/14/2022

RANDOM VARIABLES

A random variable is a measurable function X from a sample space S to $[-\infty, \infty]$.

$$X : S \to [-\infty, \infty]$$

Measurable function means that

$P(X \leq x)$ is defined for every $x \in [-\infty, \infty]$.

The function $F : [-\infty, \infty) \longrightarrow [0,1]$ where $F(x) = P(X \leq x)$

is called the cumulative distribution.

A basic example of a random variable coming from our football discussion of odds:

P(Michigan football wins in a game against Northwestern) $= 0.8$

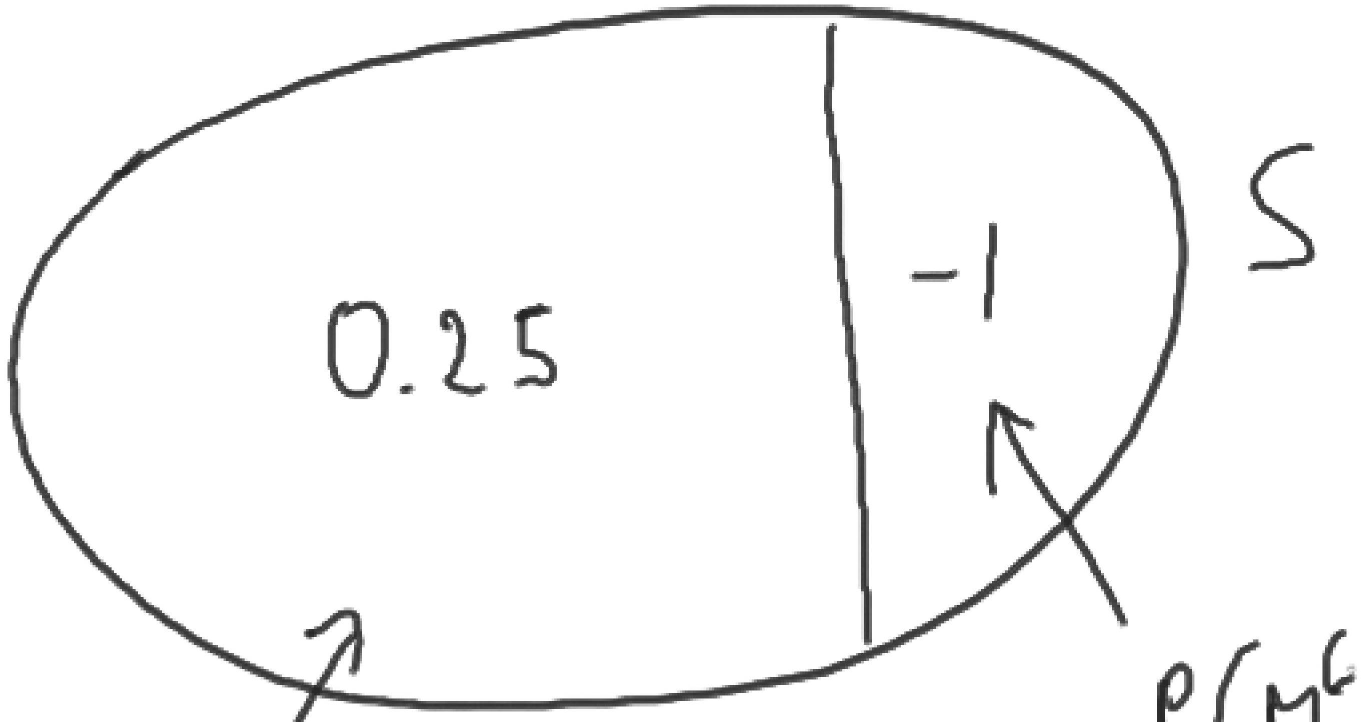
X = Winnings if I bet at odds $4:1$ on Michigan to win

$P(X < x)$ is also defined. Why?

$$[-\infty, x) = \bigcup_{n \in \mathbb{N}} \left[-\infty, x - \frac{1}{n}\right]$$

X

Suppose I bet \$1.



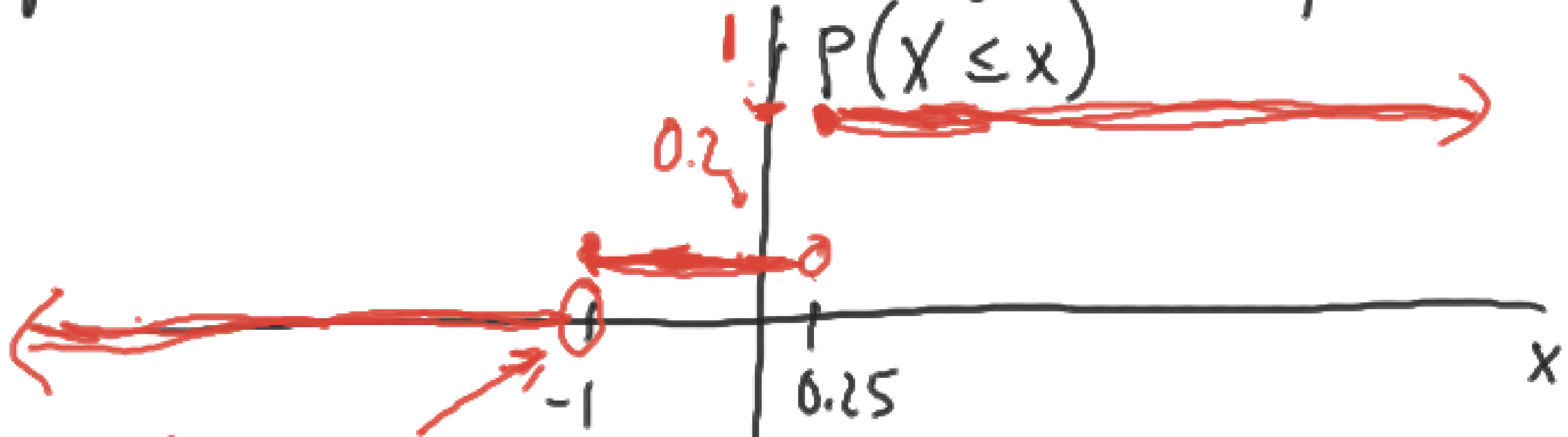
Michigan wins

$P(M) = 0.8$

$P(M^c) = 0.2$

possible values of the random variable

Graph the cumulative distribution of X:



open circle means not included

We notice: in this example, the graph is constant, except that it "jumps" at the two points $x = -1$, $x = 0.25$

This is an example of a <u>discrete random variable</u> which means that it takes on at most countably many values.

For a discrete random variable (such as in the example) it suffices to just know the probability that a given value occurs:

$$P(X = x) \qquad \text{when } x \in \{\text{values}\}$$

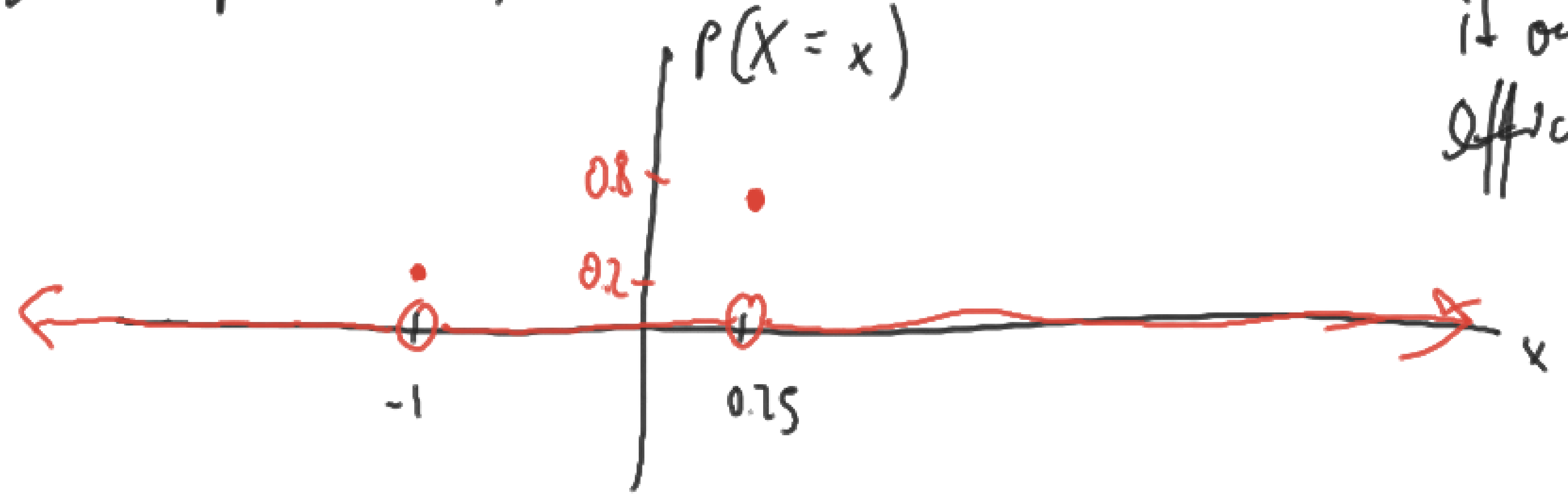
This is called the <u>probability mass function</u>.

In the example of the winning in the bet of $\$1$ on the Michigan/Northwestern game:

$$P(X=-1) = 0.2 \qquad P(X=0.25) = 0.8$$

these add up to 1.

Graphing the probability mass function:



May writing it out is more efficient than the graph?

What do the cumulative distribution (or the probability mass function in the discrete case)

say about the random variable?

statistics

Everything we can measure about it!

Do I know the random variable if I know its distribution?

NO.

Different random variables can have the same distribution, e.g. I could bet at something else at the same odds.

Scope of jointly distributed random variables

same sample space

Relation to sample space reduction.

If I have a discrete random variable X on a sample space S considering its distribution amounts to reducing the sample space S to the set of values of X.

countable

What kind of statistics may we be interested in?

① Expectation: $$E(X) = \sum_{\text{values } x} x \cdot P(X = x).$$

In the example of betting on the Michigan game:

$$P(X=-1)=0.2 \qquad P(X=0.25)=0.8$$

$$E(X) = -1\cdot 0.2 + 0.25\cdot 0.8 = \underline{\underline{0}}.$$

(a fair bet: if I bet many times, I will break even).

Probability of Michigan winning is still 80%

Follow-up: Suppose the betting company only pays 20¢ on a \$1 bet the Michigan to win. How much do I expect to win or lose on a \$1 bet?

Solution: Random variable Y

$$P(Y=-1)=0.2 \qquad P(Y=0.2)=0.8$$

$$E(Y) = -1 \cdot 0.2 + 0.2 \cdot 0.8 = -0.2 + 0.16 = \underline{\underline{-0.04}}$$

Answer: I expect to lose 4¢.

HW Due Wed 10/19 10 AM.

8. Suppose I have a discrete random variable X with

$$P(X=-2) = 0.3,\quad P(X=0) = 0.2,\quad P(X=1) = 0.5.$$

a) Graph the cumulative distribution of X (mind which boundaries are included!)

b) Graph the probability mass function of X.

c) Calculate $E(X)$.