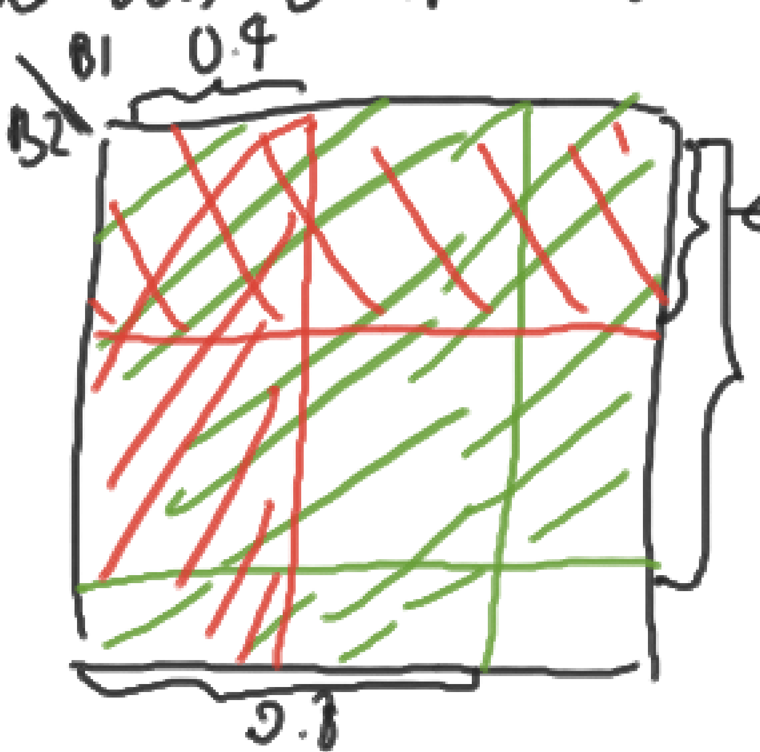
MATH 425 10/31/2022

Review

Conditional probability: A large bin contains balls, 80% of which are green and 50% have a red dot (these are independent events). A friend has chosen 2 balls at random and told me that he has a green ball with a red dot. What is the probability that the other ball he has chosen is also green?

Answer

$$\frac{48}{64} = \underline{\underline{\frac{3}{4}}}$$



$0.8 \cdot 0.5 = 0.4$

0.8 $P(E \cap F) =$

$0.32 + 0.32 - 0.4 \cdot 0.4$
$= 0.48$

$0.8 \cdot 0.4 = 0.32$

$P(F) = 0.4 + 0.4 - 0.4 \cdot 0.4$
$= 0.64$

Conditional sample space (at least one green ball with red dot)

Bayes formula: A risk factor has probability 0.6.

If a subject has it, their probability of an accident is 0.5

If a subject doesn't have it, their probability of an accident is 0.2

(a) Suppose the subject had an accident. What is the probability they had the risk factor?

(b) Suppose the subject didn't have an accident. What is the probability they didn't have the risk factor?

$P(R) = 0.6$, $P(A|R) = 0.5$

$P(A|R^c) = 0.2$

$\therefore P(R^c) = 0.4$

(a) $$\frac{0.5 \cdot 0.6}{\underbrace{0.5 \cdot 0.6}_{0.3} + \underbrace{0.2 \cdot 0.4}_{0.08}} = \frac{30}{38}$$

(b) $$\frac{0.8 \cdot 0.4}{0.8 \cdot 0.4 + 0.5 \cdot 0.6} = \frac{32}{62}$$

Multiple scenario case:

3 lightbulbs:	T1	T2	T3
frequency:	30%	20%	50%
P(last 100 hours):	20%	40%	80%

If a lightbulb lasted 100 h, what is the probability it was of T1?

$$\frac{0.2\cdot 0.3}{\underbrace{0.2\cdot 0.3 + 0.4\cdot 0.2 + 0.8\cdot 0.5}_{P(\text{lasts 100 hours})}} = \dots$$

relevant summand (T1)

<u>Odds</u> Yankees beat the Red Sox with probability 60%

How much should I offer on a \$1 bet for Yankees to win or the Red Sox to win (to break even)?

$p = 0.6$

$$O = \frac{p}{1-p} = \frac{0.6}{0.4} = 1.5$$

offer on Red Sox to win

opposite odds: reciprocal $\frac{0.4}{0.6} = 0.66$

on Yankees to win

Discrete random variables statistics

$$P(X=1)=0.2 \quad P(X=-1)=0.5, \quad P(X=2)=0.3$$

? $E(X)$, $var(X)$, $\sigma(X)$.

$$E(X)=0.2\cdot 1+0.5\cdot(-1)+0.3\cdot 2 = 0.3$$

$$E(X^2)=0.2\cdot 1+0.5\cdot 1+0.3\cdot 4 = 1.9$$

$$var(X)=E(X^2)-E(X)^2 = 1.9-0.09 = 1.81$$

$$\sigma(X) = \sqrt{1.81}$$

Binomial random variable

$X_{n,p}$ = the distribution of a sum of n independent Bernoulli variables with probability p each.

$$P(X_{n,p} = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$E(X_{n,p}) = np \qquad \mathrm{var}(X_{n,p}) = np(1-p)$$

$$\sigma(X_{n,p}) = \sqrt{np(1-p)}$$

Binomial variables occur in problems with replacement:

100 balls, 60 red, 40 green. Draw a ball 5 times, record color, throw back.

(a) P of getting exactly 3 red?

(b) average number of red ones?

(c) standard dev.?

Binomial $n = 5$, $p = \frac{60}{100} = \frac{3}{5}$

(a) $\binom{5}{3} \cdot \left(\frac{3}{5}\right)^3 \cdot \left(\frac{2}{5}\right)^2$

(b) $5 \cdot \frac{3}{5} = 3$

(c) $= \sqrt{5 \cdot \frac{3}{5} \cdot \frac{2}{5}} = \sqrt{\frac{6}{5}}$

Hypergeometric - scenario w/o replacement. N balls m are marked n drawn $X^N_{m,n}$ = number of marked ball drawn.

$$P(\underbrace{X^N_{m,n}}=k) = \frac{\binom{m}{k}\binom{N-m}{n-k}}{\binom{N}{n}}$$

$$\parallel$$

$$P(X^N_{n,m}=k)$$

$$\underline{\underline{E(X^N_{m,n}) = \frac{mn}{N}}}$$

Similar example as before: 100 balls 60 red 40 green draw 5 balls,

hyp. thm.

ⓐ $P(3\text{ red}) : \boxed{\frac{\binom{60}{3}\cdot\binom{40}{2}}{\binom{100}{5}}}$

ⓑ Average # of red: $\frac{60\cdot 5}{100} = \underline{\underline{3}}$

hypergeometric
$N = 100$
$m = 60$
$n = 5$

Poisson distribution

Binomial $n >> 0$, $\lambda = \underline{\underline{np}}$ constant

$$P(Y_\lambda = k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

$$E(Y_\lambda) = \lambda$$

$$var(Y_\lambda) = \lambda$$

$$\sigma(Y_\lambda) = \sqrt{\lambda}$$

$0! = 1$

Call center: average # of calls / hour = 3

average # of calls / 2h = 6

$\underbrace{P(\text{at least one call in 2 hours})?}$

Y_6

$\underline{1 - e^{-6}}$

Different call center: P(at least one call in an hour) $= 0.5$

? average # of calls in 4 hours ?

4 · average # of calls in an hour

$P(Y_\lambda = 0) = 0.5$

$= e^{-\lambda}$

$\lambda = -\ln(0.5)$

Answer: $\underline{\underline{-4\ln(0.5)}}$

Negative binomial

$N_{r,p}$ = # of trials

repeated (independent) Bernoulli trials with probability of "success" p
stop after "succeeding" r times

$$P(N_{r,p} = n) = \binom{n-1}{r-1} p^r (1-p)^{n-r}$$

$$E(N_{r,p}) = \frac{r}{p}$$

$$\mathrm{Var}(N_{r,p}) = \frac{r(1-p)}{p^2}$$

$$\sigma(N_{r,p}) = \frac{\sqrt{r(1-p)}}{p}$$

Device is replaced after 5 failures
P failure each time turned on: 0.2
How many times will the device be turned on?

$N_{5,0.2}$

① P(turned on exactly 20 times) =

$n=20$

$$= \boxed{\binom{19}{4} \cdot 0.2^5 \cdot 0.8^{15}}$$

② Average number of times the device will serve: $\frac{5}{0.2} = \underline{\underline{25}}$

③ $\sigma = \frac{\sqrt{5 \cdot 0.8}}{0.2} = \frac{2}{0.2} = \underline{\underline{10}}$