MATH 425 11/21/2022

The central limit theorem says that if a random variable is a sum of a large number of independent random variables with the same distribution, then it is approximately normally distributed.

<u>Example</u> (binomial) A standard cubic die is cast <u>80 times</u>. What ^many is the probability that 6 comes up at least 20 times?

<u>Solution</u>: Approximately normal:

X = # times 6 comes up

binomial: $n = 80$, $p = 1/6$

$E(X) = 80/6$

$\sigma(X) = \dfrac{\sqrt{80 \cdot 5}}{6} = \dfrac{20}{6} = \dfrac{10}{3}$

$\dfrac{X - 80/6}{10/3} \sim Z$

we should have used 19.5

$P\left(Z > \dfrac{20 - 80/6}{10/3}\right) = P(Z > 2) = 1 - P(Z < 2) = 1 - 0.9772 = \underline{0.0228}$

Example (Negative binomial) The probability of a hitter getting a hit in one at bat is 0.3. What is the probability that they will score at most 6 hits in 20 at bats?

use normal approximation to negative binomial.

we should have used 19.5

Solution: $P(N_{6,0.3} \geq 20)$

(You could also use the binomial distribution $= P(X_{20,0.3} \leq 6)$)

$E(N_{6,0.3}) = \frac{6}{0.3}$ $\quad \sigma(N_{6,0.3}) = \frac{\sqrt{6\cdot 0.7}}{0.3}$

$P\left(Z \geq \frac{20 - 6/0.3}{\sqrt{4.2}/0.3}\right) = \underline{\underline{1/2}}$

0

The Poisson distribution also becomes approximately normal when $\lambda > 0$. $E(Y_\lambda) = \lambda$, $\sigma(Y_\lambda) = \sqrt{\lambda}$.

$$\boxed{\frac{Y_\lambda - \lambda}{\sqrt{\lambda}} \approx Z}$$

<u>Example</u>: A customer support center gets about 100 phone calls per hour. What is the probability it will get more than 110 phone calls in a given hour?

<u>Solution</u>: $\lambda = 100$, $\sqrt{\lambda} = 10$

$$P\left(Z > \frac{110.5 - 100}{10}\right) = P(Z > 1.05) = 1 - 0.8531 = \underline{\underline{0.1469}}.$$

Statistical tests

Scenario: Suppose we know the distribution of some random variable X in the general population. (normal is most common). Can we "prove" that an individual is not a member of the general population. We measure the value x. If $P(X < x) \geq 0.95$ then we conclude that the individual is not in the general population. Otherwise, we say nothing.

	$X \geq x$	$X < x$
G (General population)	5%	95%
not general population	?	?

wrong: probability 5%

What is the probability of me as a scientist being wrong on this research?

Example:

The IQ-test has distribution $Z_{100,15}$. Suppose a subject is called "gifted" if there IQ test has percentile $\geq 95\%$. What score would they have to get to be labelled as "gifted"?

Solution:

$$z = \frac{Z_{100,15} - 100}{15}$$

$$P(Z < z) \approx 0.95 \qquad z \approx 1.65$$

$$\frac{Z_{100,15} - 100}{15} = 1{,}65 \qquad Z_{100,15} > 100 + 15 \cdot 1.65 \approx \underline{\underline{124.75}}$$

HW 1 The yield of a typical investment fund in a certain year is normally distributed with expectation 8% and standard deviation 2%. A hedge fund yielded 12% that year. Can we conclude with 95% certainty that the hedge fund has some non-standard ability?

2 There are an average of 240 crimes in a certain city per year. What is the probability there will be at least 250 crimes in a given year?

use the 0.5 correction!

3 What is the probability that 6 comes up at most 35 times when we cast a standard cubic die 200 times?