MATH 417 1/6/2023

Bretscher: Linear algebra (any edition)

Syllabus:
- Matrices
- systems of linear equations
- linear transformations
- Vector spaces
- Determinants
- Eigenvalues and Eigenvectors ← complex numbers
- Diagonalizing matrices, Jordan form (beyond diagonalisation)

- Orthogonal matrices (linear transformations which preserve distance)

- Principal axes (spectral theorem), singular values

(- other topics)

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3 Midterms in class in person. Tentative dates:
(no final) Feb 8, Mar 15, Apr 17.
Worth 100 pts. each

HW: graded by percentage worth 100

On Gradescope. Access code 575EZE (any problems ⟶ email me ikriz@umich.edu)

HW assigned in class, posted (typed) on gradescope on Friday
due the following class after Friday (usually Monday 10 AM)
<u>LATE HW NOT ACCEPTED</u>!

Office: GH 3846   Office Hour: MWF $\underbrace{\text{11:00 - 12:00}}$ (in person) or
by appointment.

Notes: on my Webpage. (google Igor Kriz) under Teaching
(updated today)

Matrices = rectangular arrays of numbers:

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

real matrix: a_{ij} are real numbers

$m \times n$ matrix

m rows, n columns

Example:

$$\begin{pmatrix} 2 & -1 & 3.1 \\ 12 & 0 & \pi \end{pmatrix}$$

2×3 matrix

2 rows

$$\begin{pmatrix} 2 & -1 & 3.1 \\ 12 & 0 & \pi \end{pmatrix}$$

3 columns

Algebraic operations with matrices:

$$\begin{pmatrix} 2 & -1 & 3.5 \\ 12 & 0 & 2 \end{pmatrix} + \begin{pmatrix} 4 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 6 & 0 & 4.5 \\ 13 & 2 & 2 \end{pmatrix}}}$$

(add at each pt)

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} + \begin{pmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & & \vdots \\ b_{m1} & \cdots & b_{mn} \end{pmatrix} = \begin{pmatrix} a_{11}+b_{11} & \cdots & a_{1n}+b_{1n} \\ \vdots & & \vdots \\ a_{m1}+b_{m1} & \cdots & a_{mn}+b_{mn} \end{pmatrix}$$

$$3\begin{pmatrix} 2 & -1 & 3.5 \\ 12 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 6 & -3 & 10.5 \\ 36 & 0 & 6 \end{pmatrix}$$

Multiplication of matrices:

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \cdot \begin{pmatrix} 4 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2\cdot 4 + 1\cdot 1 & 2\cdot 1 + 1\cdot(-1) \\ 3\cdot 4 + 2\cdot 1 & 3\cdot 1 + 2\cdot(-1) \end{pmatrix} = \begin{pmatrix} 9 & 1 \\ 14 & 1 \end{pmatrix}$$

mark rows   mark columns   dot products

$$\begin{pmatrix} x_1 & \cdots & x_m \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix} = x_1 y_1 + x_2 y_2 + \cdots + x_n y_n \quad \Big\} \circledast$$

(in this example, $m = n$)

row vector = $1 \times m$ - matrix   column vector = $m \times 1$ - matrix

A KEY POINT!

When can we multiply matrices?

$$A \cdot B = C$$

$m \times n$ - matrix $\qquad$ $n \times p$ - matrix $\qquad$ an $m \times p$ - matrix

the same so we can form the dot-product $\circledast$

For example:

same length

$$\begin{pmatrix} 2 & 1 & 4 \\ -1 & 0.5 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 12 \\ -1 \end{pmatrix}$$

Yes, makes sense.

$2 \times 3 \qquad = \qquad 3 \times 1 \qquad 2 \times 1$

do not match

$$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 & 4 \\ -1 & 0.5 & 1 \end{pmatrix}$$

does not make sense!

Quick summary: order of multiplication of matrices matters, sometimes it does not make sense

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HW due Monday 1/9 10 AM

1 Decide which of the following matrix multiplications make sense, and when they do, calculate the answer:

a $\begin{pmatrix} 2 & 4 \\ 1 & 1 \\ 3 & 5 \end{pmatrix} \cdot \begin{pmatrix} -1 & 2 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}$

b $\begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 4 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 1 & 4 \\ 1 & 3 & 1 & 1 \end{pmatrix}$

(c) $$\begin{pmatrix} 1 \\ 2 \\ 1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 & -1 & 3 \end{pmatrix}$$

(2) Find an example of 2×2 matrices A, B such that

$$AB \neq BA.$$