MATH 417 2/24/2023

<u>Example</u>: Find the linear approximation of the mapping $f: \mathbb{R}^4 \to \mathbb{R}^2$ given by

$$f\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} x^2 - yt + xz \\ xt + y^2 z \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix}$$

dependent variables

independent variables

at the point $\begin{pmatrix} x_0 \\ y_0 \\ z_0 \\ t_0 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \end{pmatrix}$.

<u>Solution</u>:

$$f\begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1^2+1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Jacobi matrix:

$$\begin{pmatrix} \partial u/\partial x & \partial u/\partial y & \partial u/\partial z & \partial u/\partial t \\ \partial v/\partial x & \partial v/\partial y & \partial v/\partial z & \partial v/\partial t \end{pmatrix} = \begin{pmatrix} 2x+z & -t & x & -y \\ t & 2yz & y^2 & x \end{pmatrix} = \begin{pmatrix} 3 & 0 & 1 & 1 \\ 0 & -2 & 1 & 1 \end{pmatrix}$$

$$\boxed{f\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \sim \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 & 0 & 1 & 1 \\ 0 & -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} x-1 \\ y+1 \\ z-1 \\ t \end{pmatrix}}$$

<u>Answer</u>.

Determinants

$\det(A)$ $\qquad A = \underbrace{n \times n \text{ matrix}}_{\text{square}}$

A 2×2: $\quad \det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$

The determinant of an $n \times n$ matrix is the sum of diagonal products with certain signs.

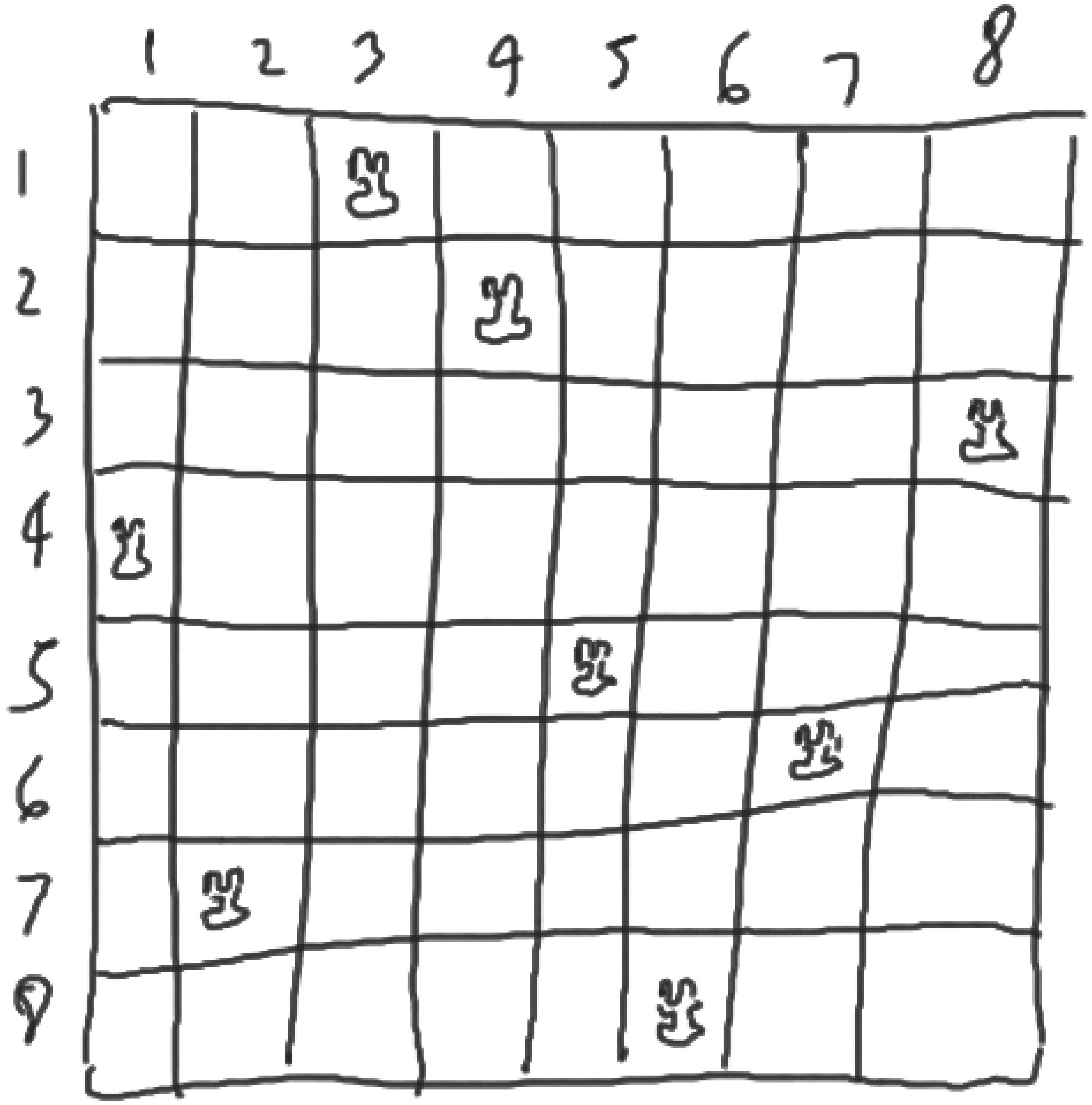
in the right sense

♖ ← chess rook

moves in rows or columns

Generalised diagonals (used in the determinant) are arrangements of a maximal number of rooks on an $n \times n$ chessboard so they don't check each other (in each row and each column, there is exactly one).

$A = (a_{ij})$



The product corresponding to the arrangement:

$a_{13}\, a_{24}\, a_{38}\, a_{41}\, a_{55}\, a_{67}\, a_{72}\, a_{86}$

$(3\ 4\ 8\ 1\ 5\ 7\ 2\ 6)$ abbreviated notation (second coordinate only) (first goes in order)

$\{1, 2, 3, 4, 5, 6, 7, 8\}$

$\underbrace{\{1, 2, 3, 4, 5, 6, 7, 8\}}$

This is a bijective mapping

$\sigma : \{1,2,3,4,5,6,7,8\} \to \{1,2,3,4,5,6,7,8\}$

$\sigma(1) = 3 \quad \sigma(2) = 4 \quad \sigma(3) = 8 \quad \sigma(4) = 1 \quad \sigma(5) = 5$

$\sigma(6) = 7 \quad \sigma(7) = 2$

Such a σ is called a <u>permutation</u>.

In permutation notation, we can write the product as:

$a_{1\sigma(1)}\, a_{2\sigma(2)} \cdots a_{n\sigma(n)}$

How many permutations $\sigma : \{1, \dots, n\} \to \{1, \dots, n\}$ are there?

<u>Answer</u>: $n! = n(n-1)(n-2) \cdot \dots \cdot 1$

(For $\sigma(1)$, we have n choices. For $\sigma(2)$, we have $(n-1)$ choices. . .)

$$\det(A) = \sum_{\sigma \text{ permutations}} \text{sign}(\sigma)\, a_{1\sigma(1)} a_{2\sigma(2)} \cdots a_{n\sigma(n)}.$$

$A = (a_{ij})$

The sign of a permutation is the most subtle point.

How to get the sign of a permutation?

The rule: switching two numbers switches the sign.

<u>Example</u>: Calculate sign(34815726)

<u>Solution</u>:

3 4 8 1 5 7 2 6

1

1 4 8 3 5 7 2 6

2

1 2 8 3 5 7 4 6

3

1 2 3 8 5 7 4 6

4

1 2 3 4 5 7 8 6

5

1 2 3 4 5 6 8 7

6

1 2 3 4 5 6 7 8

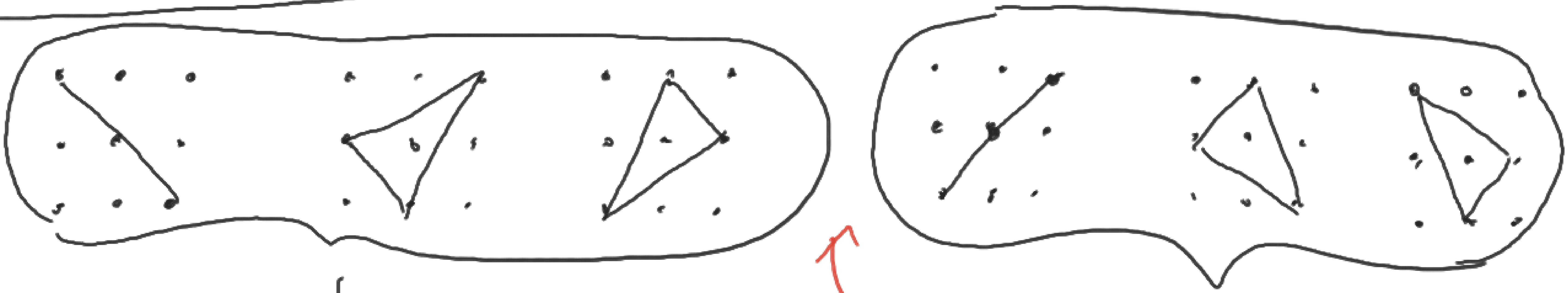
Strategy (each time get one number where it belongs)

6 switches even number:

<u>Answer</u>: +

sign(34815726) = +

3×3 determinants: $3! = 6$



$+$ $\qquad\qquad$ $-$

Example: Calculate $\det\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 3 & 1 & 2 \end{pmatrix} =$

$$= 2 + 6 + 6 - 9 - 8 - 1 = \underline{\underline{-4}} \quad \text{Answer}$$

order of terms as above

HW

5. Calculate $\operatorname{sign}(2\,3\,7\,1\,4\,6\,5)$.

6. Calculate $\det\begin{pmatrix} 3 & 2 & 1 \\ 1 & 1 & -5 \\ 4 & 1 & 1 \end{pmatrix}$.

(Show your work!)

Due 3/6 10 AM.